Proceedings of DETC'03
2003 ASME Design Engineering Technical Conferences and Computers and Information in Engineering Conference
Chicago, Illinois USA, September 2-6, 2003

DETC2003/DAC-48818

DIMENSIONAL SYNTHESIS OF CRR SERIAL CHAINS

Alba Perez
Robotics and Automation Laboratory
Department of Mechanical and Aerospace Engineering
University of California
Irvine, California 92612
Email: maperez@uci.edu

J.M. McCarthy*
Robotics and Automation Laboratory
Department of Mechanical and Aerospace Engineering
University of California
Irvine, California 92612
jmmccart@uci.edu

ABSTRACT

This paper presents the kinematic synthesis of a CRR serial chain. This is a four-degree-of-freedom chain constructed from a cylindric joint and two revolute joints in series. The design equations for this chain are obtained from the dual quaternion kinematics equations evaluated at a specified set of task positions. In this case, we find that the chain is completely defined by seven task positions. Furthermore, our solution of these equations has yielded 52 candidate designs, so far. There may be many more. This synthesis methodology shows promise for the design of constrained serial chains.

INTRODUCTION

In this paper, our focus is on the geometric design of serial robots that have less than six degrees of freedom, which we term *constrained robots*. The design of robotic systems has generally focussed on serial chains with six or more degrees-of-freedom. [Chedmail, 1998] and [Gosselin, 1998], present optimization techniques for design of serial and parallel robotic systems that provide the desired properties of the workspace. In the kinematic synthesis of robots with six or more degrees of freedom, the only specification from a displacement point of view is that the desired task lies inside the boundaries of the six-dimensional volume defined by the link lengths and joint limits. In this case, the design concern is how to reach the goal positions, and the instantaneous kinematics parameters are the main focus of the design, see [Kumar and Waldron, 1981]. However, in the synthesis of constrained robots, the main problem is the definition of the boundaries of the workspace so that it contains the desired task. This can be viewed as an extension of the kinematic synthesis of linkages ([McCarhty, 2000]).

Linkage Synthesis

Spatial linkage synthesis uses the geometric properties of a serial chain to formulate algebraic equations that must be satisfied at each of a discrete set of positions in the workspace ([Suh and Radcliffe, 1978]). This yields algebraic equations that are solved to determine the dimensions of the chain, also see [McCarthy, 2000]. Examples of this are the synthesis of spatial RR chains ([Tsai and Roth, 1973], [Perez, 2000]), spatial CC chains ([Chen, 1969], [Huang, 2000]) and SS chains ([Innocenti, 1994], [Liao, 2001]).

Recently, new methods have been developed which use the kinematics equations of the robot to create the design equations. [Larochelle, 2000] uses planar quaternions to define an approximate synthesis for planar robots. Mavroidis and Lee used the kinematics equations of the spatial RR and RRR robots to formulate their design equations. This approach introduces the joint parameters of the chain at each of the goal positions as additional variables ([Mavroidis, 2001], [Lee, 2002]). The advantage is that it can be systematically applied to a broad range of robotic systems.

*Address all correspondence to this author.

Overview

In this paper we use the kinematics equations of the robot expressed as successive screw displacements ([Gupta, 1986], [Tsai, 1999]) and formulated in terms of dual quaternions. Dual quaternions were introduced to linkage analysis by [Yang and Freudenstein, 1964]. They form an eight dimensional Clifford algebra that contains a subset, known as unit dual quaternions, which is isomorphic to the group of spatial displacements ([McCarthy, 1990]). Also see [Angeles, 1998].

THE CRR SERIAL CHAIN

The CRR robot is a special case of the RPRR serial chain in which the direction of the axes of the initial revolute (R) and prismatic (P) joints are parallel. Let the fixed axis $\mathsf{G} = \mathbf{g} + \varepsilon\mathbf{g}^0$ allow a rotation by θ and be connected to an axis $\mathsf{H} = \mathbf{h} + \varepsilon\mathbf{h}^0$ that allows the translation d. If we require the directions $\mathbf{h}$ and $\mathbf{g}$ to be parallel then the combination forms a cylindric (C) joint (Figure 1)—notice that the axes G and H need not be coincident. The third joint is a rotation of angle ϕ about an axis $\mathsf{W} = \mathbf{w} + \varepsilon\mathbf{w}^0$, and the fourth joint is a rotation ψ about an axis $\mathsf{F} = \mathbf{f} + \varepsilon\mathbf{f}^0$, see Figure 1.

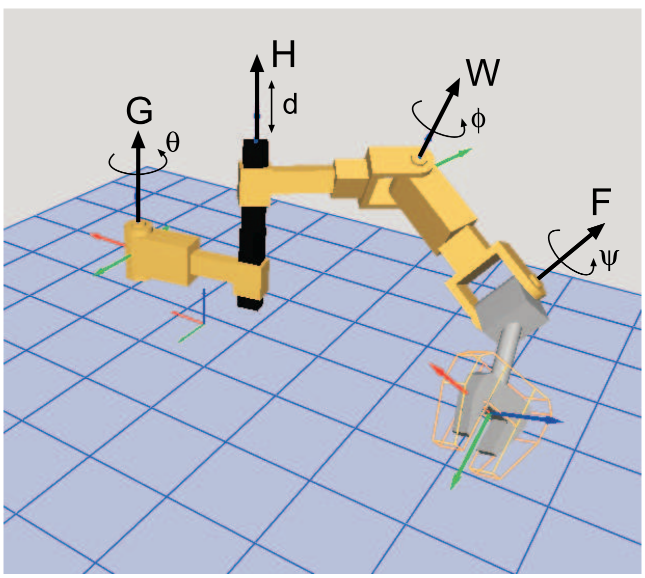

Figure 1. The CRR robot

THE KINEMATICS EQUATIONS

The kinematics equations of the robot equate the 4×4 homogeneous transformation $[D]$ between the end-effector and base frame to the sequence of local coordinate transformations along the chain (Craig 1989),

$$[D] = [G][Z(\theta_1, d_1)][X(\alpha_{12}, a_{12})][Z(\theta_2, d_2)]\dots [X(\alpha_{n-1,n}, a_{n-1,n})][Z(\theta_n, d_n)][H]. \quad (1)$$

The parameters (θ, d) define the movement at each joint and (α, a) are the length and twist of each link, collectively known as the Denavit-Hartenberg parameters. The transformation $[G]$ defines the position of the base of the chain relative to the fixed frame, and $[H]$ locates the tool relative to the last link frame.

These kinematics equations can be transformed into relative displacements choosing a reference position $[D_0]$. We then compute $[D_{0i}] = [D_i][D_0]^{-1}$, that is

$$[D_{0i}] = [D_i][D_0]^{-1} = ([G][Z(\theta_{1i}, d_{1i})]\dots[Z(\theta_{ni}, d_{ni})][H])([G][Z(\theta_{10}, d_{10})]\dots[Z(\theta_{n0}, d_{n0})][H])^{-1}. \quad (2)$$

This can be viewed as successive screw displacements along the joint axes S_i from the reference configuration [Perez and McCarthy, 2002],

$$[D_{0i}] = [T(\Delta\theta_1, \mathsf{S}_1)]\dots[T(\Delta\theta_n, \mathsf{S}_n)]. \quad (3)$$

In order to create the design equations, we transform the matrix kinematics equations to dual quaternion kinematics equations. This is a useful formulation from the synthesis point of view because the components of each axis appear explicitly in the base frame coordinates.

A spatial displacement can be represented as a dual quaternion $\hat{Q}(\hat{\theta}) = \sin(\frac{\hat{\theta}}{2})\mathsf{S} + \cos(\frac{\hat{\theta}}{2})$, where $\mathsf{S} = \mathbf{s} + \varepsilon\mathbf{s}^0$, with $\varepsilon^2 = 0$, is the screw axis of the transformation. The dual numbers $\cos(\frac{\hat{\theta}}{2}) = \cos\frac{\theta}{2} + \varepsilon(-\frac{d}{2}\sin\frac{\theta}{2})$ and $\sin(\frac{\hat{\theta}}{2}) = \sin\frac{\theta}{2} + \varepsilon(\frac{d}{2}\cos\frac{\theta}{2})$ contain the information about the rotation and the displacement along the screw axis.

The dual quaternion form for the kinematics equations of the robot are obtained by transforming eq.(3) into

$$\hat{D}^i = \hat{S}_1(\Delta\hat{\theta}^i_1)\dots\hat{S}_n(\Delta\hat{\theta}^i_n), \quad (4)$$

where $\hat{D}^i$ is the dual quaternion for $[D_{0i}]$ and $\hat{S}_j(\Delta\hat{\theta}^i_j)$ is the dual quaternion for $[T(\Delta\theta_j, \mathsf{S}_j)]$.

The kinematics equations of a general RPRR serial chain in dual quaternion form are

$$\hat{Q}_{RPRR}(\theta, d, \phi, \psi) = \hat{G}(\theta, 0)\hat{H}(0, d)\hat{W}(\phi, 0)\hat{F}(\psi, 0), \quad (5)$$

and for the case when the direction of H is the same as the direction of G, the kinematics equations simplify to

$$\hat{Q}_{CRR}(\theta, d, \phi, \psi) = \hat{G}(\theta, d)\hat{W}(\phi, 0)\hat{F}(\psi, 0). \tag{6}$$

DESIGN EQUATIONS

Let $\hat{Q}(\hat{\theta}_1, \ldots, \hat{\theta}_k)$ be the kinematics equations of a serial robot in dual quaternion form, and let a discrete approximation of the desired workspace be given in the form of n goal dual quaternions $\hat{P}^i, \quad i = 1, \ldots, n-1$. The synthesis problem consists of solving the $n-1$ vector equations

$$\hat{Q}^i(\hat{\theta}^i_1, \ldots, \hat{\theta}^i_k) = \hat{P}^i, \quad i = 1, \ldots, n-1. \tag{7}$$

For each of the $n-1$ positions we have eight component equations. However, due to the structure of the dual quaternions, only six of them are independent.

The structure of the chain determines how many arbitrary positions we can fit in its workspace; the concrete values of the task positions define in turn its geometric dimensions. We compute the maximum number of positions by counting the number of equations and unknowns in Eq.(7). We assume that the axes of the rotational and translational joints are not related, but it is possible to adapt the formula to other cases just by adding the required constraints. Every revolute joint axis consists of four parameters that define a line in space; prismatic joints consist of two variables that define the direction of the slide. In addition, we need the value of the joint variables to reach each task position.

Having in mind that in a general displacement, orientations operate separately from translations, we can define the maximum number of orientations n_R and the maximum number of complete positions n_{max}. For a robot chain represented by a series of r revolute joints and t prismatic joints, the maximum number of complete positions is given by

$$n_{max} = \frac{3r+t+6}{6-r-t}, \tag{8}$$

and counting only the maximum number of orientations, we obtain

$$n_R = \frac{3+r}{3-r}. \tag{9}$$

The cases in which $n_R < n_{max}$, we will be able to define only n_R arbitrary complete positions, and the rest will be translational components of dual quaternions in which the orientation will have to be bounded to the given workspace. Notice that a necessary condition for this to happen is that the number of revolute joints of the robot must be at most two.

The CRR robot can reach, applying Eq.(8), $n_{max} = 7$ complete arbitrary positions. The design equations are

$$\hat{Q}_{CRR}(\theta^i, d^i, \phi^i, \psi^i) = \hat{P}^i, \quad i = 1, \ldots 6 \tag{10}$$

We call this the *parameterized design equations*, because they contain the joint variables θ, d, ϕ and ψ. The complete workspace of the robot can be defined by giving values to these variables in the kinematics equations. We can solve the parameterized equations as they appear in Eq.(10), in which case we solve not only for the dimensions of the robot, but also for the values of the joint variables to reach the goal positions. As a set of equations we can use the whole eight components of each dual quaternion, or use the six independent components plus the Plucker constraints for each axis,

$$\begin{aligned}
&Q^i_{CRR} = \begin{Bmatrix} q_x + \varepsilon q_{x0} \\ q_y + \varepsilon q_{y0} \\ q_z + \varepsilon q_{z0} \end{Bmatrix}^i = \begin{Bmatrix} p_x + \varepsilon p_{x0} \\ p_y + \varepsilon p_{y0} \\ p_z + \varepsilon p_{z0} \end{Bmatrix}^i, \quad i = 1, \ldots 6, \\
&\mathbf{g} \cdot \mathbf{g} = 1, \quad \mathbf{g} \cdot \mathbf{g}_0 = 0, \\
&\mathbf{w} \cdot \mathbf{w} = 1, \quad \mathbf{w} \cdot \mathbf{w}_0 = 0, \\
&\mathbf{f} \cdot \mathbf{f} = 1, \quad \mathbf{f} \cdot \mathbf{f}_0 = 0.
\end{aligned} \tag{11}$$

We prefer to use this second set because the Plucker constraints are usually simpler than the dual quaternion equations. The design problem, using the parameterized equation for the CRR chain, yields a set of 42 equations in 42 unknowns.

We can also eliminate the joint variables from the equations to obtain a set of *reduced design equations*. Doing so we reduce the dimension of the problem by $(r+t)(n-1)$ variables, which means 24 variables in the case of the CRR chain. This procedure is advantageous when it eventually leads to an algebraic solution for the design equations; otherwise the elimination of the joint variables usually makes the equations more complicated.

Reduced Design Equations

To create the reduced design equations, we eliminate the joint variables to solve only for the axis variables, which define the physical dimensions of the robot.

In order to eliminate the joint parameters, we consider the equations for each position separately. The methodology is presented here for the CRR serial chain; for more details see [Perez and McCarthy, 2002b]. We solve linearly for up to three joint variables and use the relations among those joint variables to define the reduced design equations.

The parameterized design equations in Eq.(10) can be written as the linear transformation

$$\hat{Q}_{CRR}(\theta,d,\phi,\psi) = [\hat{M}]\hat{V}(\theta,d,\phi) = \hat{P}. \qquad (12)$$

where $\hat{V}$ is the dual quaternion

$$\hat{V}(\theta,d,\phi) = \begin{Bmatrix} \sin\frac{\theta}{2}\cos\frac{\phi}{2} + \varepsilon\frac{d}{2}\cos\frac{\theta}{2}\sin\frac{\phi}{2} \\ \cos\frac{\theta}{2}\sin\frac{\phi}{2} + \varepsilon\frac{d}{2}\sin\frac{\theta}{2}\cos\frac{\phi}{2} \\ \sin\frac{\theta}{2}\sin\frac{\phi}{2} + \varepsilon\frac{d}{2}\cos\frac{\theta}{2}\cos\frac{\phi}{2} \\ \cos\frac{\theta}{2}\cos\frac{\phi}{2} + \varepsilon\frac{d}{2}\sin\frac{\theta}{2}\sin\frac{\phi}{2} \end{Bmatrix}. \qquad (13)$$

If we write the dual quaternions as 8-dimensional vectors, the matrix for the coordinate robot of Eq.(13) has the form:

$$[\hat{M}] = \begin{bmatrix} A & \vdots & 0 \\ \dots & & \dots \\ B & \vdots & C \end{bmatrix}. \qquad (14)$$

We solve for the components of the vector $\hat{V}$ by inverting the matrix $[\hat{M}]$,

$$\hat{V}(\theta,d,\phi) = \begin{bmatrix} A^{-1} & \vdots & 0 \\ \dots & & \dots \\ -C^{-1}BA^{-1} & \vdots & C^{-1} \end{bmatrix}\hat{P}, \qquad (15)$$

to obtain, for the rotational components of $\hat{V}$,

$$\begin{Bmatrix} \sin\frac{\theta}{2}\cos\frac{\phi}{2} \\ \cos\frac{\theta}{2}\sin\frac{\phi}{2} \\ \sin\frac{\theta}{2}\sin\frac{\phi}{2} \\ \cos\frac{\theta}{2}\cos\frac{\phi}{2} \end{Bmatrix} = [A]^{-1}\hat{P}_R, \qquad (16)$$

and for the translational components,

$$\begin{Bmatrix} \frac{d}{2}\cos\frac{\theta}{2}\sin\frac{\phi}{2} \\ \frac{d}{2}\sin\frac{\theta}{2}\cos\frac{\phi}{2} \\ \frac{d}{2}\cos\frac{\theta}{2}\cos\frac{\phi}{2} \\ \frac{d}{2}\sin\frac{\theta}{2}\sin\frac{\phi}{2} \end{Bmatrix} = -([C]^{-1}[B][A]^{-1})\hat{P}_R + [C]^{-1}\hat{P}_D, \qquad (17)$$

where $\hat{P}_R$, $\hat{P}_D$ are the real and dual part of the goal dual quaternion, respectively. The solutions are functions of the joint variable ψ. The relations among the variables we solved for in Eq.(16) and Eq.(17) give us the solution for ψ and allow us to create two reduced design equations per position, R_1 and R_2, which are free of joint variables.

These two reduced design equations, plus the set of Plucker constraints, form the final set of 18 reduced equations in 18 parameters,

$$\begin{aligned} &\{R_1, R_2\}^i, \quad i = 1, \dots 6, \\ &\mathbf{g}\cdot\mathbf{g} = 1, \quad \mathbf{g}\cdot\mathbf{g}_0 = 0, \\ &\mathbf{w}\cdot\mathbf{w} = 1, \quad \mathbf{w}\cdot\mathbf{w}_0 = 0, \\ &\mathbf{f}\cdot\mathbf{f} = 1, \quad \mathbf{f}\cdot\mathbf{f}_0 = 0. \end{aligned} \qquad (18)$$

NUMERICAL EXAMPLE

The goal positions for the following example have been randomly generated and are shown in Figure 1. They are specified as screw axis, rotation and translation in Table 1.

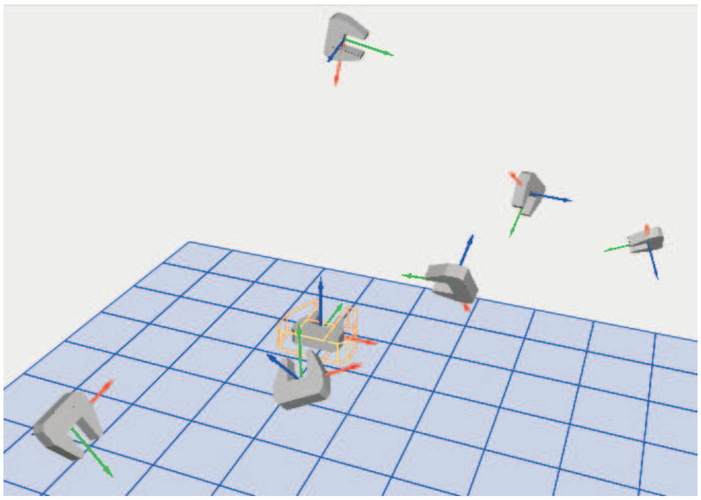

Figure 2. The seven goal positions

We solved the parameterized design equations using a Newton-Raphson solver (fsolve) in MatLab on a MacG4 (733MHz). Starting with random initial conditions, we obtained 52 different real solutions from 1000 runs of 3000 iterations each. Each run took approximately 15 seconds; in whole, the process took about 4 hours.

The solution of the reduced design equations yielded eight real solutions following the same solution strategy, that is random initial conditions, 1000 runs of 3000 iterations each. The increased complexity of the equations required 14 hours to complete the process. Thus, numerical solution of the original parameterized equations seems more efficient. However, it may be that the reduced equations can be further simplified and optimized.

Table 1. THE GOAL POSITIONS

Pos.	*Axis*	*Rot.*	*Transl.*
1	$(1.0,0.0,0.0)+\varepsilon(0.0,0.0,0.0)$	0	0
2	$(0.66,-0.62,0.43)+\varepsilon(1.36,-0.77,-3.21)$	2.89	−2.04
3	$(-0.65,-0.68,0.33)+\varepsilon(-0.37,0.11,-0.51)$	2.33	2.90
4	$(0.50,0.68,-0.53)+\varepsilon(-2.05,1.06,-0.57)$	2.31	0.37
5	$(-0.49,0.46,0.74)+\varepsilon(1.01,-0.06,0.71)$	1.58	0.15
6	$(-0.57,0.39,-0.73)+\varepsilon(-1.48,0.87,1.58)$	2.79	0.43
7	$(0.75,-0.51,0.43)+\varepsilon(-0.68,-0.95,0.06)$	0.75	0.83

We present numerical data of 10 of the solutions in Table 2, for those chains with shortest total link length, defined as the sum of the Denavit-Hartenberg parameters $a_{12}+d_2+a_{23}$. Figure 3 presents solution 11 reaching the seven positions and Figure 5 presents snapshots of solution 4 while moving along a trajectory defined by the seven positions. The trajectory was created using the sofware SYNTHETICA 1.0, ([Su et al., 2002]).

CONCLUSIONS

This paper presents a solution to the dimensional synthesis equations for the CRR serial chain. The dual quaternion kinematics equations of this chain provide a set of parameterized design equations that can be solved efficiently using existing numerical routines. A reduced set of design equations is obtained by eliminating the joint parameters. However, the increased complexity of the reduced equations requires more time for solution, by approximately one order of magnitude.

The computer-aided-design of constrained serial chains, such as the CRR chain, requires fast algorithms that yield all the candidate designs for a given task. This is an on-going research challenge.

ACKNOWLEDGMENT

The authors gratefully acknowledge the support of the National Science Foundation.

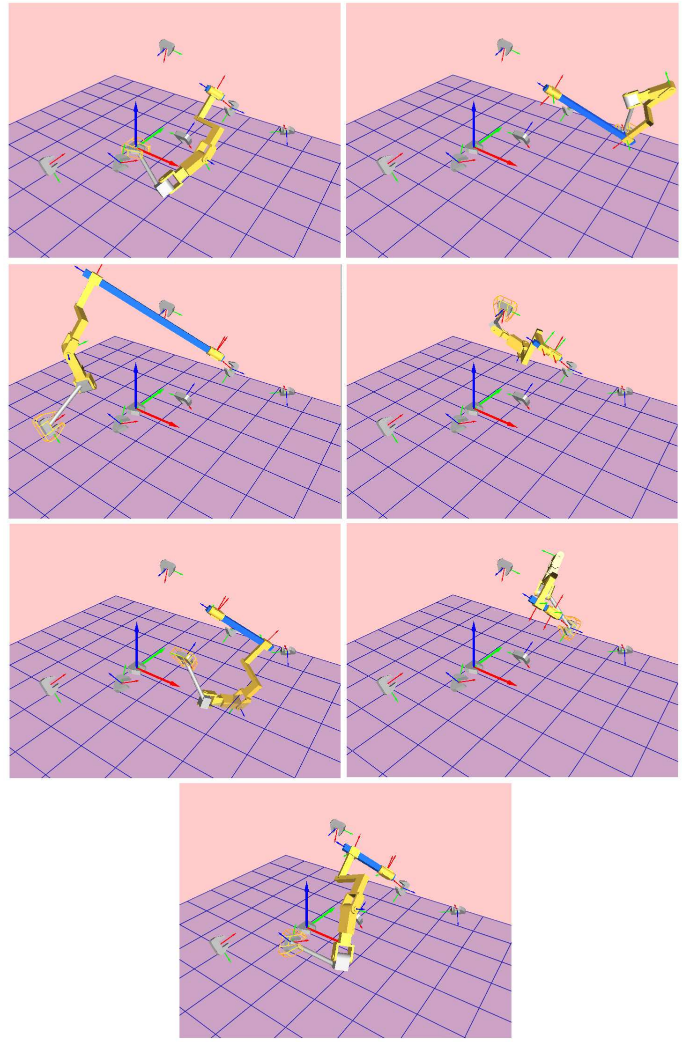

Figure 3. CRR robot reaching the seven goal positions

REFERENCES

[Angeles, 1998] Angeles, J., 1998, "The application on dual algebra to kinematic analysis," *Computational Methods in Mechanical Systems, NATO ASI Series* (ed. J. Angeles and E. Zakhariev) Springer, Berlin.

[Bottema, 1979] Bottema, O., and Roth, B., 1979, *Theoretical Kinematics*, North Holland. (reprinted Dover Publications 1990).

[Chedmail, 1998] Chedmail, P., 1998, "Optimization in Multi-DOF Mechanisms," *Computational Methods in Mechanical Systems, NATO ASI Series* (ed. J. Angeles and E. Zakhariev) Springer, Berlin.

[Chen, 1969] Chen, P., and Roth, B., 1969, Design Equations for the Finitely and Infinitesimally Separated Position Synthesis of Binary Links and Combined Link Chains, *ASME J. Eng. Ind.* **91**(1):209219.

[Cox, 1998] Cox, D., Little, J. and O'Shea, D., 1998, *Using Algebraic Geometry*, Springer, New York.

[Craig, 1986] Craig, J., 1986, *Introduction to Robotics*, Addison-Wesley.

[Gosselin, 1998] Gosselin, C. M., 1998, "On the design of efficient parallel mechanisms," *Computational Methods in Me-*

chanical Systems, *NATO ASI Series* (ed. J. Angeles and E. Zakhariev) Springer, Berlin.

[Gupta, 1986] Gupta, K.C., 1986, "Kinematic Analysis of Manipulators Using Zero Reference Position Description", *Int. J. Robot. Res.*, **5**(2):5-13

[Huang, 2000] Huang, C., and Chang, Y.-J., 2000, Polynomial Solution to the Five-Position Synthesis of Spatial CC Dyads via Dialytic Elimination, *Proceedings of the ASME Design Technical Conferences*, September 1013, 2000, Baltimore MD, Paper Number DETC2000/MECH-14102.

[Innocenti, 1994] Innocenti, C., 1994, "Polynomial Solution of the Spatial Burmester Problem." *Mechanism Synthesis and Analysis*, *ASME DE* vol. 70.

[Kumar and Waldron, 1981] Kumar, A., and Waldron, K. J., 1981, "The Workspaces of a Mechanical Manipulator," *ASME Journal of Mechanical Design*, **103**:665-672.

[Larochelle, 2000] Larochelle, P., 2000, "Approximate motion synthesis via parametric constraint manifold fitting," *Advances in Robot Kinematics* (eds. J. Lenarcic and M. M. Stanisic) Kluwer Acad. Publ., Dordrecht.

[Mavroidis, 2001] Mavroidis, C., Lee, E. and Alan, M. 2001, "A New Polynomial Solution to the Geometric Design Problem of Spatial R-R Robot Manipulators Using the Denavit and Hartenberg Parameters," *ASME J. of Mechanical Design*, **123**(1):58-67.

[Lee, 2002] Lee, E., and Mavrodis, D., 2002, "Solving the Geometric Design Problem of Spatial 3R Robot Manipulators Using Polynomial Homotopy Continuation," *ASME J. of Mechanical Design*, **124**(4):652-661.

[Liao, 2001] Liao, Q. and McCarthy, J.M., 2001, "On the seven position synthesis of a 5-SS platform linkage", *ASME J. of Mechanical Design*, **123**(1):74-79.

[McCarthy, 1990] McCarthy, J. M., 1990, *An Introduction to Theoretical Kinematics*, MIT Press.

[McCarhty, 2000] McCarthy, J. M., 2000, *Geometric Design of Linkages*, Springer, New York.

[McCarthy, 2000] McCarthy, J. M., 2000, Mechanisms Synthesis Theory and the Design of Robots, *Proceedings of the 2000 IEEE International Conference on Robotics and Automation*, April 2428 2000, San Francisco, CA.

[Perez, 2000] Perez, A., and McCarthy, J. M., 2000, Dimensional Synthesis of Spatial RR Robots, Advances in Robot Kinematics, Lenarcic, J., ed., Piran-Portoroz, Slovenia, June 2630, 2000.

[Perez and McCarthy, 2002] Perez, A., McCarthy, J. M., and Bennett, B., 2002, "Dual Quaternion Synthesis of Constrained Robots" Advances in Robot Kinematics, Lenarcic, J.,and Thomas, F., ed., Caldes de Malavella, Spain, June, 2002.

[Perez and McCarthy, 2002b] Perez, A., McCarthy, J.M., 2002, "Dual Quaternion Synthesis of a 2-TPR Constrained Parallel Robot". *Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators*, Quebec, October 2002.

[Su et al., 2002] Su, H., Collins, C., and McCarthy, J. M., "An Extensible Java Applet for Spatial Linkage Synthesis", *Proc. ASME Des. Eng. Technical Conferences*, Montreal, Canada, 2002.

[Suh and Radcliffe, 1978] Suh, C.H. and Radclie, C.W.,1978, *Kinematics and mechanisms design.* John Wiley & Sons, 1978.

[Tsai, 1999] Tsai, L.W., 1999, *Robot Analysis*. John Wiley and Sons, New York.

[Tsai and Roth, 1973] Tsai, L.W. and Roth, B., 1973, "A Note on the Design of Revolute-Revolute Cranks". *Mechanism and Machine Theory*, **8**:23-31.

[Yang and Freudenstein, 1964] Yang, A.T., and Freudenstein, F., 1964, "Application of Dual-Number Quaternion Algebra to the Analysis of Spatial Mechanisms", *ASME Journal of Applied Mechanics*, June 1964, pp.300-308.

Table 2. THE SOLUTIONS

Length	*Joint*	*Line*
1.66	G	$(-0.45,-0.14,-0.88)+\varepsilon(-1.91,0.59,0.89)$
	W	$(0.25,-0.89,0.37)+\varepsilon(1.40,-0.87,-3.03)$
	F	$(-0.88,-0.23,-0.42)+\varepsilon(-0.96,0.69,1.64)$
1.75	G	$(0.14,0.98,0.11)+\varepsilon(0.14,-0.02,-0.01)$
	W	$(-0.61,-0.60,0.51)+\varepsilon(0.79,-0.20,0.72)$
	F	$(0.44,-0.89,0.12)+\varepsilon(-0.59,-0.50,-1.55)$
2.59	G	$(-0.73,-0.07,-0.68)+\varepsilon(-1.50,0.32,1.60)$
	W	$(0.29,-0.82,-0.50)+\varepsilon(-1.14,1.54,-3.22)$
	F	$(-0.49,0.399,-0.78)+\varepsilon(-1.88,2.14,2.26)$
2.65	G	$(0.39,0.90,-0.18)+\varepsilon(-0.55,0.025,-1.08)$
	W	$(-0.42,0.54,-0.73)+\varepsilon(-1.45,0.40,1.14)$
	F	$(0.74,0.13,-0.66)+\varepsilon(-0.66,0.55,-0.63)$
2.68	G	$(0.31,0.95,0.02)+\varepsilon(0.51,-0.16,-0.26)$
	W	$(0.71,0.60,-0.37)+\varepsilon(0.01,-0.40,-0.62)$
	F	$(0.62,-0.61,-0.49)+\varepsilon(-2.15,-0.35,-2.29)$
2.99	G	$(0.31,0.95,0.02)+\varepsilon(0.51,-0.16,-0.26)$
	W	$(0.71,0.60,-0.37)+\varepsilon(0.01,-0.40,-0.62)$
	F	$(0.62,-0.61,-0.49)+\varepsilon(-2.15,-0.35,-2.29)$
3.00	G	$(-0.12,-0.98,-0.17)+\varepsilon(-1.32,0.25,-0.46)$
	W	$(0.88,0.28,0.37)+\varepsilon(1.60,-2.84,-1.63)$
	F	$(0.26,0.487,-0.83)+\varepsilon(-1.45,0.67,-0.06)$
3.06	G	$(0.18,-0.94,-0.27)+\varepsilon(-0.76,0.32,-1.64)$
	W	$(0.98,-0.15,-0.09)+\varepsilon(-0.39,-1.93,-0.89)$
	F	$(-0.20,-0.55,0.81)+\varepsilon(0.51,-0.76,-0.39)$
3.08	G	$(0.05,-0.69,-0.71)+\varepsilon(-2.22,0.25,-0.40)$
	W	$(-0.35,-0.25,-0.90)+\varepsilon(-2.75,2.09,0.47)$
	F	$(0.90,0.36,-0.25)+\varepsilon(-0.07,-0.50,-0.99)$
3.20	G	$(-0.146,-0.77,-0.62)+\varepsilon(-2.67,0.17,0.41)$
	W	$(-0.90,0.41,-0.15)+\varepsilon(0.89,2.10,0.42)$
	F	$(-0.68,-0.09,0.72)+\varepsilon(2.09,0.59,2.04)$

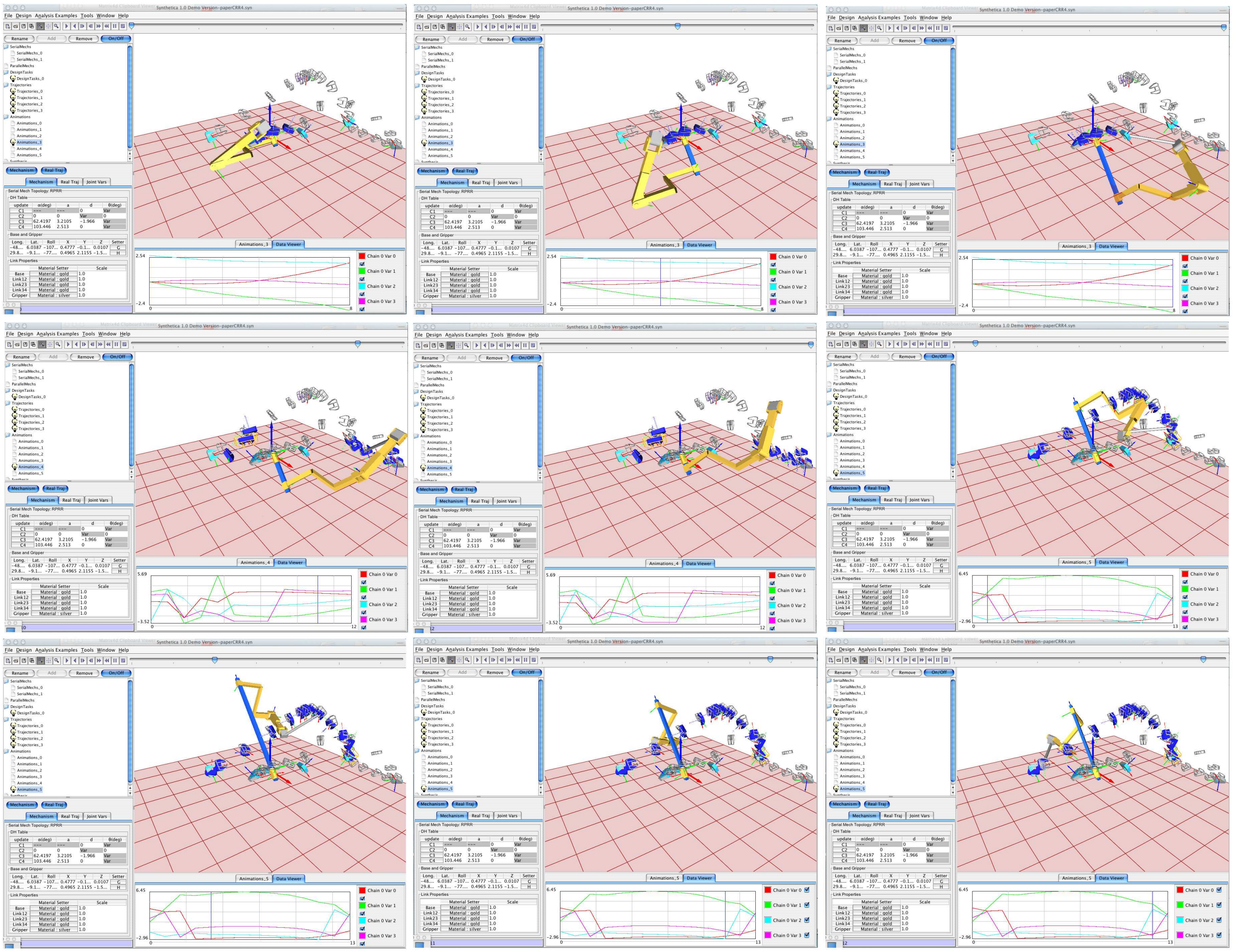

Figure 4. CRR robot moving along a trajectory defined by the seven goal positions