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Dimensional Synthesis of RPC Serial Robots

Alba Perez and **J. Michael McCarthy**
Robotics and Automation Laboratory
Dept. of Mechanical and Aerospace Engineering
University of California, Irvine
Irvine, CA 92697
maperez@uci.edu, jmmccarthy@uci.edu

Abstract

This paper presents a synthesis methodology for the RPC serial robot. This robot has a four dimensional configuration space that can be generated by specifying a finite set of goal positions. The methodology uses the kinematics equations of the robot in dual quaternion form evaluated at each of the goal positions to form design equations. The structure of these design equations allows a systematic elimination of the joint parameters and a solution for the dimensions of the RPC robot. An example is provided to demonstrate the theory.

1 Introduction

This paper presents a new formulation for the kinematic synthesis of constrained robots. A constrained robotic system is one in which each supporting chain imposes a kinematic constraint on the workpiece. These systems provide structural support in certain directions while allowing freedom of movement in others. Our synthesis methodology uses a set of goal positions that describe the workspace of the constrained robot. The dual quaternion kinematic equations of the chain are evaluated at each goal position to obtain the design equations. The solutions of these are the physical dimensions of the robot.

This synthesis methodology is an extension of the kinematic synthesis of linkages, McCarthy (2000b), which is based on finding the geometric constraints of the serial chain. The advantage of an approach based on the expression of the kinematic equations is that it can be applied systematically to serial chains with up to five degrees of freedom. Multiple solutions obtained with this method can be combined to create a parallel robot.

2 Literature Review

Spatial linkage synthesis uses the geometric properties of a serial chain to formulate algebraic equations that must be satisfied at each of a discrete set of positions in the workspace, Suh and Radcliffe (1978). This yields algebraic equations that are solved to determine the dimensions of the chain. Also see McCarthy (2000). Examples of this are the synthesis of spatial RR chains (Tsai and Roth (1973), Perez and McCarthy (2000)), spatial CC chains (Chen and Roth (1969), Huang and Chang (2000)) and SS chains (Innocenti (1994), Liao and McCarthy (1998)).

Recently, Mavroidis and Lee (2001) used the kinematics equations of the spatial RR and RRR robots to formulate their design equations. This approach introduces the joint parameters of the chain at each of the goal positions as additional variables in the design equations, see also Lee and Mavroidis (2002). The advantage is that it can be systematically applied to a broad range of robotic systems.

We use successive screw displacements (Gupta (1986), Tsai (1999)), formulated in terms of dual quaternions, to represent the kinematics equations of the robot. Dual quaternions were introduced to linkage analysis by Yang and Freudenstein (1964). They form an eight dimensional Clifford algebra that contains a subset, known as unit dual quaternions, which is isomorphic to the group of spatial displacements, McCarthy (1990). Also see Angeles (1998). This formulation has been applied to several spatial robots, Perez and McCarthy (2002).

There are two advantages in this formulation. The first is that successive screw displacements provide a convenient formulation for the kinematics equations in terms of the joint axes directly. Secondly, it reduces the number of equations obtained in each goal position from 12 to 8.

3 Kinematics Equations

The kinematics equations of the robot equate the 4×4 homogeneous transformation $[D]$ between the end-effector and the base frame to the sequence of local coordinate transformations along the chain (Craig (1986)),

$$[D] = [G][Z(\theta_1, d_1)][X(\alpha_{12}, a_{12})][Z(\theta_2, d_2)] \dots \\ \dots [X(\alpha_{n-1,n}, a_{n-1,n})][Z(\theta_n, d_n)][H]. \quad (1)$$

The parameters (θ, d) define the movement at each joint and (α, a) are the length and twist of each link, collectively known as the Denavit-Hartenberg parameters. The transformation $[G]$ defines the position of the base of the chain relative to the fixed frame, and $[H]$ locates the tool relative to the last link frame.

3.1 Successive Screw Displacements

These kinematics equations can be transformed into successive screw displacements by choosing a reference position $[D_0]$. Let $[D_i]$ be the homogeneous matrix describing the transformation from the fixed frame to a moving frame F_i. We can compute $[D_{0i}] = [D_i][D_0]^{-1}$, that is,

$$[D_{0i}] = [D_i][D_0]^{-1} = \\ ([G][Z(\theta_{1i}, d_{1i})] \dots [Z(\theta_{ni}, d_{ni})][H]) \\ ([G][Z(\theta_{10}, d_{10})] \dots [Z(\theta_{n0}, d_{n0})][H])^{-1}. \quad (2)$$

This can be viewed as

$$[D_{0i}] = [T(\Delta\theta_1^i, \mathsf{S}_1)] \dots [T(\Delta\theta_n^i, \mathsf{S}_n)], \quad (3)$$

where

$$[T(\Delta\theta_1^i, \mathsf{S}_1)] = [G][Z(\theta_{1i}, d_{1i})][Z(\theta_{10}, d_{10})]^{-1}[G]^{-1}, \\ [T(\Delta\theta_2^i, \mathsf{S}_2)] = ([G][Z(\theta_{10}, d_{10})][X(\alpha_{12}, a_{12})][Z(\theta_{2i}, d_{2i})]) \\ ([G][Z(\theta_{10}, d_{10})][X(\alpha_{12}, a_{12})][Z(\theta_{20}, d_{20})])^{-1}, \\ \vdots \\ [T(\Delta\theta_n^i, \mathsf{S}_n)] = ([G][[Z(\theta_{10}, d_{10})] \dots) \\ [Z(\theta_{ni}, d_{ni})][Z(\theta_{n0}, d_{n0})]^{-1}([G][[Z(\theta_{10}, d_{10})] \dots)^{-1}. \quad (4)$$

The displacements $[T(\Delta\theta_i, \mathsf{S}_i)]$ are the relative rotations about and translations along the joint axes S_i of the robot from the chosen reference configuration. Notice that by expressing them in this way, the initial transformation $[G]$ is absorbed in the first joint axis and the final transformation $[H]$ disappears from the expression.

3.2 Dual Quaternion Kinematics Equations

The workspace of the robot can also be expressed by using the Clifford algebra of the *dual quaternions*. A spatial displacement can be represented as a dual quaternion,

$$\hat{Q}(\hat{\theta}) = \sin(\frac{\hat{\theta}}{2})\mathsf{S} + \cos(\frac{\hat{\theta}}{2}), \quad (5)$$

where $\mathsf{S} = \mathbf{s} + \epsilon\mathbf{s}^0$, with $\epsilon^2 = 0$, is the screw axis of the transformation. The dual numbers $\cos(\frac{\hat{\theta}}{2}) = \cos\frac{\theta}{2} + \epsilon(-\frac{d}{2}\sin\frac{\theta}{2})$ and $\sin(\frac{\hat{\theta}}{2}) = \sin\frac{\theta}{2} + \epsilon(\frac{d}{2}\cos\frac{\theta}{2})$ contain the information about the rotation about and the displacement along the screw axis. The components of the dual quaternions can be easily computed from the homogeneous matrix transformation.

The spatial displacements can be represented as the set of points $\mathbf{Z} = (\mathbf{Z}, \mathbf{Z}^0)$ in $\mathbf{R}^8$ which are subject to two constraints: $\mathbf{Z} \cdot \mathbf{Z} = 1$ and $\mathbf{Z} \cdot \mathbf{Z}^0 = 0$.

The dual quaternion form for the kinematics equations of the robot are obtained by transforming eq.(3) into

$$\hat{D}^i = \hat{S}_1(\Delta\hat{\theta}_1^i) \dots \hat{S}_n(\Delta\hat{\theta}_n^i), \quad (6)$$

where $\hat{D}^i$ is the dual quaternion for $[D_{0i}]$ and $\hat{S}_j(\Delta\hat{\theta}_j^i)$ is the dual quaternion for $[T(\Delta\theta_j^i, \mathsf{S}_j)]$.

This approach yields the kinematics equations as successive screw transformations from the reference position. It is a useful formulation from the synthesis point of view because the components of each axis appear explicitly in the base frame coordinates.

3.3 Kinematics equations for the RPC robot

The RPC robot is a four-degree-of-freedom robot. The fixed axis $\mathsf{G} = \mathbf{g} + \epsilon\mathbf{g}^0$ allows rotation of angle θ about it. This is followed by a translation d along an arbitrary direction $\mathbf{h}$, and finally a rotation of angle ϕ about an arbitrary axis $\mathsf{W} = \mathbf{w} + \epsilon\mathbf{w}^0$ and a translation b along the same direction $\mathbf{w}$, see Figure 1.

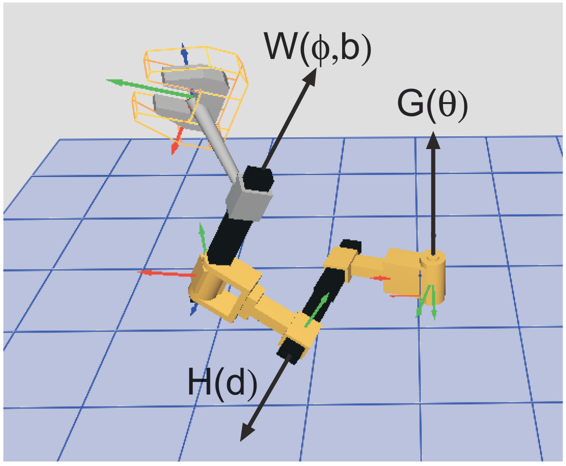

Figure 1: *The spatial RPC robot*

The dual quaternion kinematics equations are obtained by composing the dual quaternions that represent each joint axis,

$$\hat{Q}_{RPC}(\theta, d, \phi, b) = \hat{G}(\theta, 0)\hat{H}(0, d)\hat{W}(\phi, b) \quad (7)$$

4 Design Equations

4.1 Dual quaternion design equations

Let $[T(\theta_1, \dots, \theta_k)]$ be the kinematics equations of a serial robot, and let a discrete approximation of the desired workspace be given in the form of n goal transformations $[P_i], i = 0, \ldots, n-1$. The synthesis problem consists of solving the n matrix equations

$$[T(\hat{\theta}_{1,i}, \ldots, \hat{\theta}_{k,i})] = [P_i], \quad i = 0, \ldots, n-1. \tag{8}$$

We now transform these equations to successive screw displacements in dual quaternion form. Equating the $n-1$ goal positions $\hat{P}^i, i = 1, \ldots, n-1$ to the kinematics equations $\hat{Q}(\hat{\theta}_1, \ldots, \hat{\theta}_k)$, we obtain the $n-1$ equations

$$\hat{Q}_i(\hat{\theta}^i_1, \ldots, \hat{\theta}^i_k) = \hat{P}^i, \quad i = 1, \ldots, n-1. \tag{9}$$

For each of the $n-1$ positions we define eight component equations. However, due to the structure of the dual quaternions, only six of them are independent.

4.2 Counting

The workspace of a robot is an algebraic surface whose shape is defined by the structure of the robot. The shape limits the maximum number of arbitrary task positions that we can fit into it; the concrete values of the task positions define in turn the dimensions of the workspace. We need to be able to know the maximum number of task positions n_{max} that we can specify for every different robot.

The following counting is for a robot consisting of r rotational joints and t translational joints. We assume that the axes of the rotational and translational joints are not related, but it is possible to adapt the formula to other cases just by adding the required constraints. Every revolute joint axis consists of four parameters that define a line in space; prismatic joints consist of two variables that define the direction of the slide. In addition, we need the value of the joint variables to reach each one of the task positions.

We equate the number of unknowns to the number of equations, which are six for each dual quaternion equality, to obtain

$$n_{max} = \frac{3r + t + 6}{6 - r - t}. \tag{10}$$

Due to the semi-direct product structure of displacements in space, where rotations operate separately from translations, a counting formula is needed to ensure that there is no limitation of the maximum number of task orientations n_R,

$$n_R = \frac{3 + r}{3 - r} \tag{11}$$

The cases in which $n_R < n_{max}$ can be solved for only n_R complete goal positions.

4.3 Design equations for the RPC robot

The kinematics equations obtained in Eq. (7), together with the goal positions $\hat{P}^i$, are used to create the design equations

$$\hat{Q}_{RPC}(\theta^i, d^i, \phi^i, b^i) = \hat{P}^i, \quad i = 1, \ldots, n-1. \tag{12}$$

We compute the maximum number of complete positions, with $r = 2$, $t = 2$, but considering that the direction of the prismatic joint b coincides with the axis of the revolute joint ϕ. We obtain $n_{max} = 6$ complete spatial goal positions. However, if we count how many rotations we can reach with two revolute joints, we obtain $n_R = 5$. The maximum number of complete displacements that we can specify is $n = 5$. In order to avoid using partially specified positions, we impose the prismatic axis $\mathbf{h}$ to be perpendicular to both revolute axes,

$$\mathbf{g} \cdot \mathbf{h} = 0 \quad , \quad \mathbf{w} \cdot \mathbf{h} = 0. \tag{13}$$

With this extra constraint, we obtain $n_{max} = n_R = 5$.

5 Solving the Design Equations

The design equations in Eq.(9) contain joint variables and kinematic parameters defining the joint axes. Our goal is to eliminate the joint variables, if possible, and solve for the parameters of the axes, which define the physical dimensions of the robot.

In order to eliminate the joint parameters, we work with the equations corresponding to one goal position. The first step in this implicitization process uses the semi-direct product structure of the group of spatial displacements captured by the algebra of dual quaternions, which separates the composition of rotations in the real part from a combination of translations and rotations in the dual part.

The four rotational components of the dual quaternion equation are parameterized only by the revolute joint variables,

$$\hat{Q}_{rot}(\theta_1, \ldots, \theta_l) = \begin{Bmatrix} q_x(\theta_1, \ldots, \theta_l) \\ q_y(\theta_1, \ldots, \theta_l) \\ q_z(\theta_1, \ldots, \theta_l) \\ q_w(\theta_1, \ldots, \theta_l) \end{Bmatrix} = \begin{Bmatrix} p_x \\ p_y \\ p_z \\ p_w \end{Bmatrix} \tag{14}$$

This can always be transformed to a linear system that allows to solve for two of the revolute joint variables as a function of the joint axes and the rest of revolute variables,

$$[R(\theta_3, \ldots, \theta_l)] \begin{Bmatrix} \cos\frac{\theta_1}{2} \sin\frac{\theta_2}{2} \\ \sin\frac{\theta_1}{2} \cos\frac{\theta_2}{2} \\ \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2} \\ \cos\frac{\theta_1}{2} \cos\frac{\theta_2}{2} \end{Bmatrix} = \begin{Bmatrix} p_x \\ p_y \\ p_z \\ p_w \end{Bmatrix} \tag{15}$$

where the matrix $[R(\theta_3, \ldots, \theta_l)]$ is invertible for non-degenerated cases. Degenerated cases are for instance solutions in which the serial chain is not spatial but planar. We can assume the matrix is compatible when the axes are a solution for the design problem. We eliminate linearly two of the rotational parameters in the form of the vector of sine and cosines in

Eq.(15). We can then substitute these expressions in the second four components of the dual quaternion,

$$\hat{Q}_{trans}(\theta_3,\ldots,\theta_l,d_1,\ldots,d_k) = = \begin{Bmatrix} q_x^0(\theta_3,\ldots,\theta_l,d_1,\ldots,d_k) \\ q_y^0(\theta_3,\ldots,\theta_l,d_1,\ldots,d_k) \\ q_z^0(\theta_3,\ldots,\theta_l,d_1,\ldots,d_k) \\ q_w^0(\theta_3,\ldots,\theta_l,d_1,\ldots,d_k) \end{Bmatrix} = \begin{Bmatrix} p_x^0 \\ p_y^0 \\ p_z^0 \\ p_w^0 \end{Bmatrix}. \quad (16)$$

The subsequent joint variables can be eliminated sequentially in a similar fashion. The reduced set of equations consists of equations free of joint variables and conditions on the joint variables we are solving for.

To the final set of reduced equations we need to add the Plucker conditions for each joint axis $\mathsf{S}_i = \mathbf{s}_i + \epsilon \mathbf{s}_i^0$.

$$\begin{aligned} \mathbf{s}_i \cdot \mathbf{s}_i &= 1, \quad i = 1,\ldots,k+l \\ \mathbf{s}_i \cdot \mathbf{s}_i^0 &= 0, \quad i = 1,\ldots,k. \end{aligned} \quad (17)$$

These equations are in fact the ones that allow us to consider only six of the eight equations in each dual quaternion equality.

5.1 Solving the design equations for the RPC robot

We epxand Eq. (7), $\hat{Q}_{RPC} = \hat{Q}_0 + \mathsf{Q}$, to obtain

$$\begin{aligned} &\hat{Q}_0 = c\frac{\theta}{2}c\frac{\phi}{2} - \mathbf{g}\cdot\mathbf{w}s\frac{\theta}{2}s\frac{\phi}{2} + \\ &\epsilon(-(\frac{d}{2}\mathbf{g}\cdot\mathbf{h} + \frac{b}{2}\mathbf{g}\cdot\mathbf{w})s\frac{\theta}{2}c\frac{\phi}{2} - (\frac{b}{2} + \frac{d}{2}\mathbf{h}\cdot\mathbf{w})c\frac{\theta}{2}s\frac{\phi}{2} + \\ &(\frac{d}{2}(\mathbf{g}\times\mathbf{w})\cdot\mathbf{h} - (\mathbf{g}_0\cdot\mathbf{w} + \mathbf{g}\cdot\mathbf{w}_0))s\frac{\theta}{2}s\frac{\phi}{2}), \end{aligned} \quad (18)$$

and

$$\begin{aligned} &\mathsf{Q} = \mathbf{g}s\frac{\theta}{2}c\frac{\phi}{2} + \mathbf{w}c\frac{\theta}{2}s\frac{\phi}{2} + \mathbf{g}\times\mathbf{w}s\frac{\theta}{2}s\frac{\phi}{2} + \\ &\epsilon((\mathbf{g}_0 + \frac{b}{2}\mathbf{g}\times\mathbf{w} + \frac{d}{2}\mathbf{g}\times\mathbf{h})s\frac{\theta}{2}c\frac{\phi}{2} + \\ &(\mathbf{w}_0 + \frac{d}{2}\mathbf{h}\times\mathbf{w})c\frac{\theta}{2}s\frac{\phi}{2} + (\frac{b}{2}\mathbf{w} + \frac{d}{2}\mathbf{h})c\frac{\theta}{2}c\frac{\phi}{2} \\ &(\mathbf{g}_0\times\mathbf{w} + \mathbf{g}\times\mathbf{w}_0 - \frac{b}{2}\mathbf{g} + \frac{d}{2}(\mathbf{g}\cdot\mathbf{w})\mathbf{h})s\frac{\theta}{2}s\frac{\phi}{2}), \end{aligned} \quad (19)$$

where c and s stand for cosine and sine respectively.

Use the first four components to solve for the revolute joint angles θ and ϕ,

$$\begin{bmatrix} \mathbf{g} & \mathbf{w} & \mathbf{g}\times\mathbf{w} & \vec{0} \\ 0 & 0 & -\mathbf{g}\cdot\mathbf{w} & 1 \end{bmatrix} \begin{Bmatrix} \sin\frac{\theta}{2}\cos\frac{\phi}{2} \\ \cos\frac{\theta}{2}\sin\frac{\phi}{2} \\ \sin\frac{\theta}{2}\sin\frac{\phi}{2} \\ \cos\frac{\theta}{2}\cos\frac{\phi}{2} \end{Bmatrix} = \begin{Bmatrix} \mathbf{p} \\ p_w \end{Bmatrix} \quad (20)$$

to obtain

$$\begin{aligned} \sin\frac{\theta}{2}\cos\frac{\phi}{2} &= \frac{\mathbf{p}\cdot((-\mathbf{g}\times\mathbf{w})\times\mathbf{w})}{(\mathbf{g}\times\mathbf{w})\cdot(\mathbf{g}\times\mathbf{w})} \\ \cos\frac{\theta}{2}\sin\frac{\phi}{2} &= \frac{\mathbf{p}\cdot(-\mathbf{g}\times(\mathbf{g}\times\mathbf{w}))}{(\mathbf{g}\times\mathbf{w})\cdot(\mathbf{g}\times\mathbf{w})} \\ \sin\frac{\theta}{2}\sin\frac{\phi}{2} &= \frac{\mathbf{p}\cdot(\mathbf{g}\times\mathbf{w})}{(\mathbf{g}\times\mathbf{w})\cdot(\mathbf{g}\times\mathbf{w})} \\ \cos\frac{\theta}{2}\cos\frac{\phi}{2} &= \frac{\mathbf{p}\cdot(\mathbf{g}\times\mathbf{w})(\mathbf{g}\cdot\mathbf{w})}{(\mathbf{g}\times\mathbf{w})\cdot(\mathbf{g}\times\mathbf{w})} + p_w. \end{aligned} \quad (21)$$

We use the relation among these variables to create the first reduced design equation R_1 ,

$$R_1 \quad : \quad \frac{\sin\frac{\theta}{2}\sin\frac{\phi}{2}}{\cos\frac{\theta}{2}\sin\frac{\phi}{2}} = \frac{\sin\frac{\theta}{2}\cos\frac{\phi}{2}}{\cos\frac{\theta}{2}\cos\frac{\phi}{2}}. \quad (22)$$

Substitute the values of Eq. (21) in the last four design equations and eliminate d, b to obtain one additional reduced design equation, R_2.

These two reduced design equations per task position, plus the set of Plucker and extra constraints, form the final set of 15 reduced equations in 15 parameters,

$$\begin{aligned} &\{R_1, R_2\}^i, \quad i = 1,\ldots 4, \\ &\mathbf{g}\cdot\mathbf{g} = 1, \quad \mathbf{g}\cdot\mathbf{g}_0 = 0, \\ &\mathbf{w}\cdot\mathbf{w} = 1, \quad \mathbf{w}\cdot\mathbf{w}_0 = 0, \\ &\mathbf{h}\cdot\mathbf{h} = 1, \quad \mathbf{h}\cdot\mathbf{g} = 0, \quad \mathbf{h}\cdot\mathbf{w} = 0. \end{aligned} \quad (23)$$

We can always solve numerically the set of reduced design equations to obtain the parameters that define the joint axes. In the case of the RPC robot, an algebraic closed solution can be also found by solving first independently for the orientation parameters (directions $\mathbf{g}$ and $\mathbf{w}$), following for instance McCarthy (2000). The direction of the prismatic axes are defined by the extra constraints in Eq.(13), and we can solve linearly for the location of the revolute joints. We obtain that the RPC robot with the given extra constraints can have up to six solutions.

6 Numerical Example

We present an example for the design of the RPC robot. The goal displacements shown on Table 1 and Figure 2 have been randomly generated.

We solved the design equations algebraically and obtained four real solutions (no more than four real solutions have been obtained in any case). Two of them are acceptable, in the sense that the dimensions of the robot are of the same order as the task. The solutions were animated using the software SYNTHETICA 1.0, (Su et al. (2002)).

Table 2 contains the coordinates of the axes of the four real solutions. In Figure 3 we can see one of the solutions, and Figure 4 shows the two solutions while reaching the five task positions.

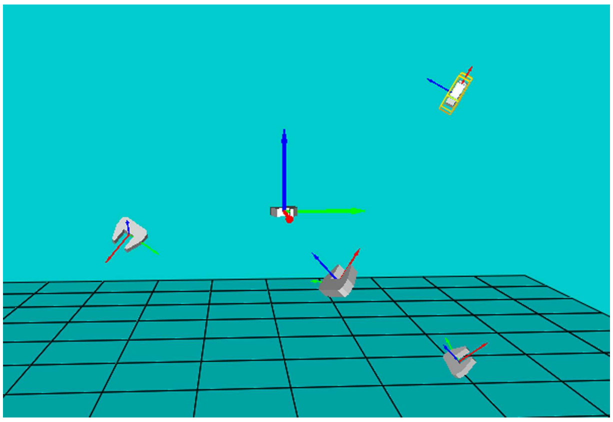

Figure 2: *The goal positions*

Axis	*Rot.*	*Trans.*
$(1.0, 0.0, 0.0; 0.0, 0.0, 0.0)$	0^o	0
$(0.33, -0.26, 0.91; 0.60, -1.02, -0.50)$	2.28	0.32
$(0.52, -0.56, 0.64; 1.10, 1.47, 0.37)$	1.43	-0.27
$(0.32, -0.84, 0.43; -0.70, 0.00, 0.52)$	5.09	1.66
$(-0.55, 0.07, -0.83; -1.31, -0.03, 0.86)$	4.55	1.09

Table 1: *The goal positions*

7 Conclusions

This paper introduces a new formulation for the kinematic synthesis of constrained parallel robots. While arbitrary serial chains can have up to six degrees of freedom, our focus is on chains with five or less degrees of freedom. These serial chains impose constraints on the workpiece of the robot. These constraints can be used to provide structural support and enhance mechanical advantage.

The dual quaternion form of the kinematics equations of the robot are evaluated at a set of goal positions to form design equations. These equations include both axis parameters that define the robot and joint parameters that define its configuration in a goal position. The structure of these equations provide a convenient strategy for the elimination of the joint parameters, which we demonstrate for the RPC robot.

Acknowledgments

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References

Angeles, J., 1998, "The application on dual algebra to kinematic analysis," *Computational Methods in Mechanical Systems, NATO ASI Series* (ed. J. Angeles and E. Zakhariev) Springer, Berlin.

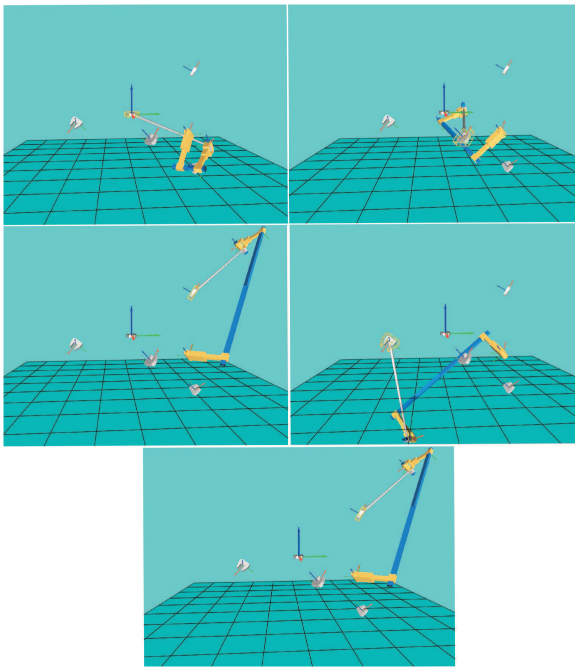

Figure 3: *The first RPC robot*

Chen, P., and Roth, B., 1969, "Design Equations for the Finitely and Infinitesimally Separated Position Synthesis of Binary Links and Combined Link Chains," *ASME J. Eng. Ind.* **91**(1), pp. 209219.

Craig, J., 1986, *Introduction to Robotics*, Addison-Wesley.

Gupta, K.C., 1986, "Kinematic Analysis of Manipulators Using Zero Reference Position Description", *Int. J. Robot. Res.*, **5**(2), pp. 5-13

Huang, C., and Chang, Y.-J., 2000, "Polynomial Solution to the Five-Position Synthesis of Spatial CC Dyads via Dialytic Elimination," *Proc. of the ASME Design Technical Conferences*, September 1013, 2000, Baltimore MD.

Innocenti, C., 1994, "Polynomial Solution of the Spatial Burmester Problem." *Mechanism Synthesis and Analysis, ASME DE* vol. 70.

Lee, E., and Mavroidis, D., 2002, "Solving the Geometric Design Problem of Spatial 3R Robot Manipulators Using Polynomial Homotopy Continuation", *ASME J. of Mechanical Design*, **124**(4), pp. 652-661.

Liao, Q. and McCarthy, J.M., 1998, "On the seven position synthesis of a 5-SS platform linkage", *ASME Journal of Mechanical Design.*

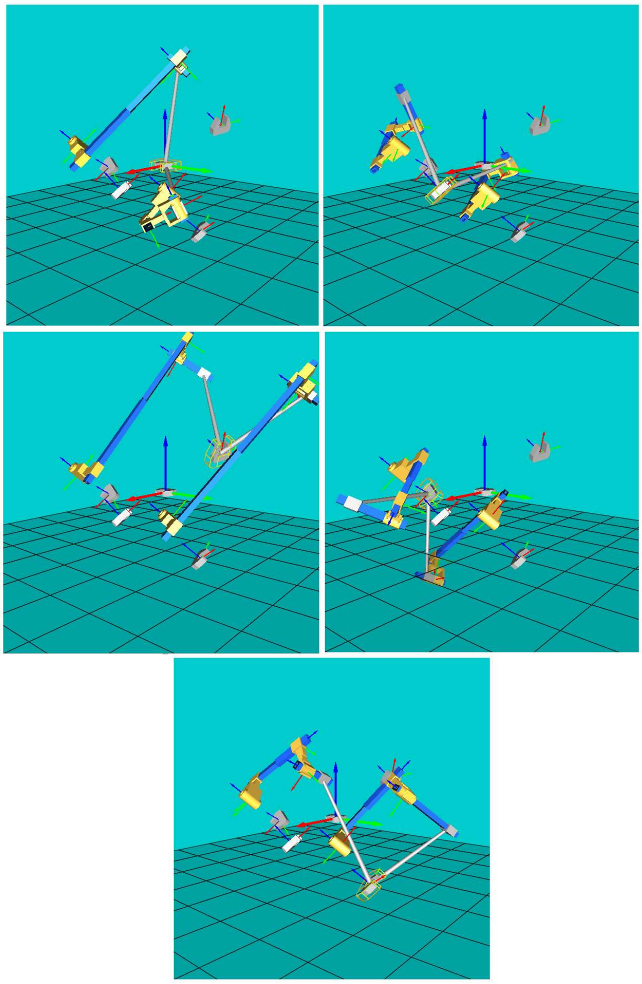

Figure 4: *The RPC solutions 1 and 2 reaching the task positions*

Joint Axis	*Direction*	*Moment*
G	$(0.51, -0.56, 0.65)$	$(0.73, -1.25, -1.64)$
H	$(-0.67, -0.73, -0.11)$	$(-0.96, 1.00, -0.85)$
W	$(0.25, -0.37, 0.89)$	$(1.50, -1.81, -1.17)$
G	$(0.69, -0.29, 0.66)$	$(-0.43, -1.22, -0.10)$
H	$(0.46, -0.53, -0.71)$	$(0.45, 2.56, -1.63)$
W	$(0.37, -0.61, 0.70)$	$(2.71, 0.53, -0.97)$
G	$(0.24, -0.79, 0.57)$	$(0.45, 1.82, 2.34)$
H	$(0.32, -0.62, -0.72)$	$(-3.02, -1.51, 2.62)$
W	$(-0.52, -0.52, 0.67)$	$(1.7, 2.81, 3.51)$
G	$(0.41, -0.82, 0.40)$	$(-14.8, 2.81, 20.8)$
H	$(-0.38, 0.25, 0.89)$	$(-15.7, 17.9, -11.7)$
W	$(0.77, 0.61, 0.16)$	$(-4.39, 5.52, 0.13)$

Table 2: *The joint axes of the RPC robot for each solution*

Mavroidis, C., Lee, E. and Alan, M. 2001, "A New Polynomial Solution to the Geometric Design Problem of Spatial R-R Robot Manipulators Using the Denavit and Hartenberg Parameters," *ASME J. of Mechanical Design* **123**(2), pp. 58-67.

McCarthy, J. M., 1990, *Introduction to Theoretical Kinematics*. The MIT Press.

McCarthy, J. M., 2000, *Geometric Design of Linkages*. Springer-Verlag.

McCarthy, J. M., 2000, "Mechanisms Synthesis Theory and the Design of Robots," *Proceedings of the 2000 IEEE International Conference on Robotics and Automation*, April 2428 2000, San Francisco, CA.

Perez, A., and McCarthy, J. M., 2000, "Dimensional Synthesis of Spatial RR Robots", Advances in Robot Kinematics, Lenarcic, J., ed., Piran-Portoroz, Slovenia, June 2630, 2000.

Perez, A., McCarthy, J. M., and Bennett, B., 2002, "Dual Quaternion Synthesis of Constrained Robots" Advances in Robot Kinematics, Lenarcic, J.,and Thomas, F., ed., Caldes de Malavella, Spain, June, 2002.

Su, H., Collins, C., and McCarthy, J. M., "An Extensible Java Applet for Spatial Linkage Synthesis", *Proc. ASME Des. Eng. Technical Conferences*, Montreal, Canada, 2002.

Suh, C.H. and Radcliffe, C.W., 1978, *Kinematics and mechanisms design.* John Wiley & Sons.

Tsai, L.W. and Roth, B., 1973, "A Note on the Design of Revolute-Revolute Cranks". *Mechanisms and Machine Theory*, 1973, Vol.8, pp 23-31.

Tsai, L.W., 1999, *Robot Analysis.* John Wiley and Sons, New York.

Yang, A.T., and Freudenstein, F., 1964, "Application of Dual-Number Quaternion Algebra to the Analysis of Spatial Mechanisms", *ASME Journal of Applied Mechanics*, June 1964, pp.300-308.