Some Open Problems in Kinematic Synthesis

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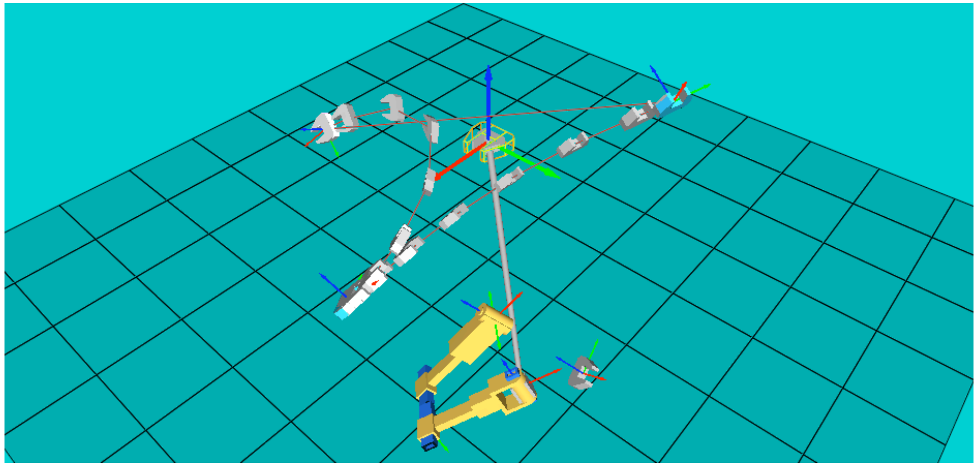
Kinematic Synthesis

Kinematic Synthesis :

- *Determine the mechanical constraints (i.e. joints) that provide a desired movement.*
- *It solves the function-to-form problem when dealing with motion.*

Finite-position Dimensional Synthesis:

- Identify a **set of task positions** that represent the desired movement of the workpiece.
- Developed for synthesis of **constrained serial open chains**. It could also be applied to parallel robots.

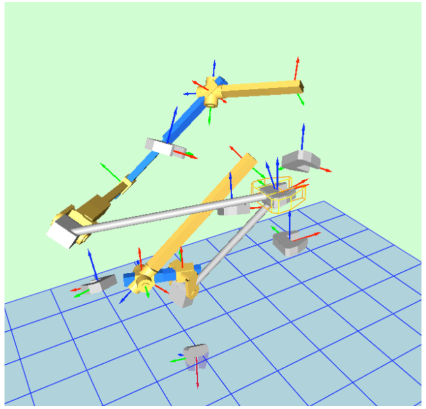


Constrained Spatial Robots

Constrained robotic system: A workpiece, or end-effector, supported by one or more serial chains such that **each one imposes at least one constraint** on its movement.

DOF	*Structure*
2	RP, RR
3	RPP, CP, RRP, RC, TP, RRR, TR
4	RPPP, CPP, RRPP, RCP, CC, TPP, RRRP, RRC, TRP, TC, SP, RRRR, TRR, TT, SR
5	RPPPP, CPPP, RRPPP, CRPP, CCP, TPPP, RRRPP, CRRP , CCR, TRPP, TCP, SPP, RRRRP , CRRR , TRRP, TTP, TCR, SRP, SC, RRRRR, TRRR, TTR, SRR, ST

Classification of constrained serial robots

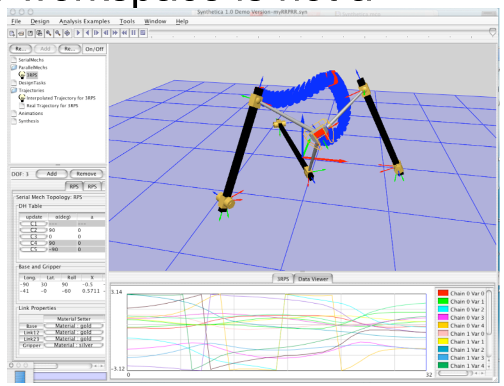


Parallel 2-TPR robot

- The constraints provide structural support in some directions, while allowing movement in the others.
- A constrained robot has less that six degrees of freedom. Its workspace is not a volume but rather a hypersurface of a certain shape.

Constraints	Assembly Categories	Total
5	5I, 3I-1II, 2I-1III, 1I-2II, 1I-1IV	487,990,859
4	4I, 2I-1II, 2II, 1I-1III, 1IV	16,734,569
3	3I, 1I-1II, 1III	464,417
2	2I, 1II	9,781
1	1I	139

Classification of constrained robotic systems

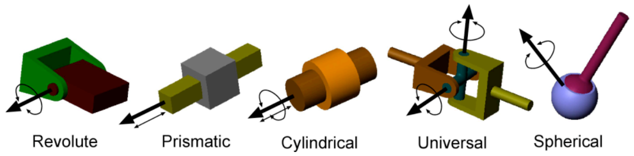


3-RPS constrained robot (category 3I, 3 degrees of freedom)

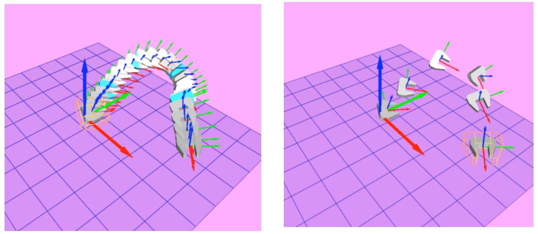
Finite-position Dimensional Synthesis

Finite-position Synthesis:

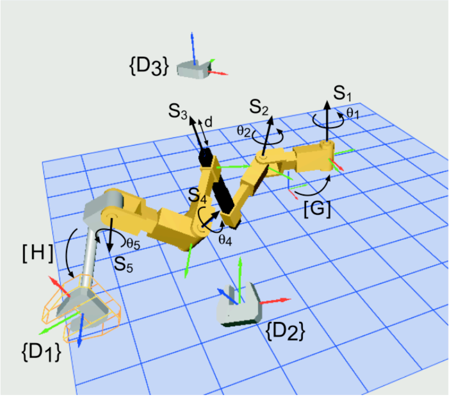
- **Given**: (a) a robot topology,



and (b) a task
(defined in terms of a set of positions
and orientations of a workpiece),



- **Find**: The location of the base, the location of the connection to the workpiece, and the dimensions of each link such the the chain reaches each task position exactly.

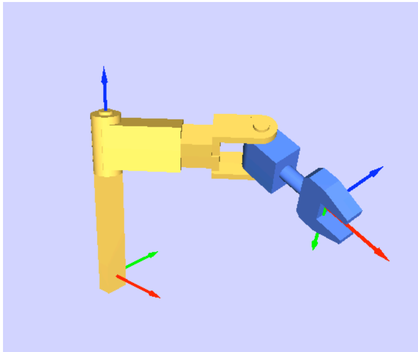


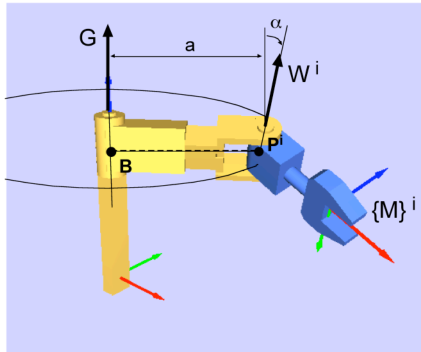
- A set of **design equations** evaluated at each of the task positions is used to determine the mechanism. There are different ways to formulate the set of design equations.

Finite-Position Dimensional Synthesis for Spatial Robots

The Design Equations for Finite Position Synthesis can be obtained in several ways:

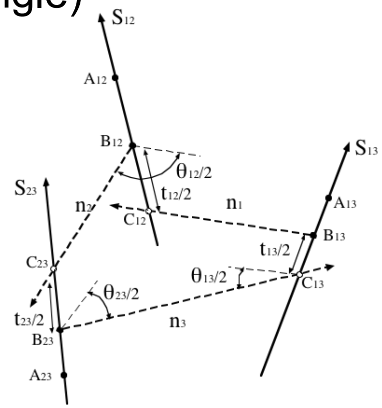
- **Geometric features** of the chain are used to formulate the algebraic constraint equations. (distance and angle constraints)





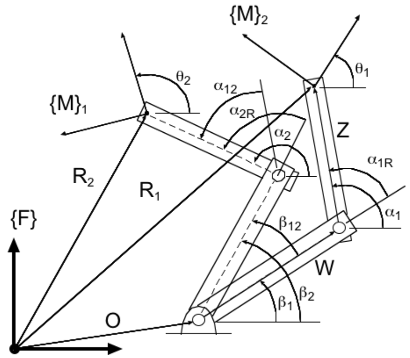
RR chain

- **Kinematic geometry** based on the screw representation of the composition of displacements. (equivalent screw triangle)

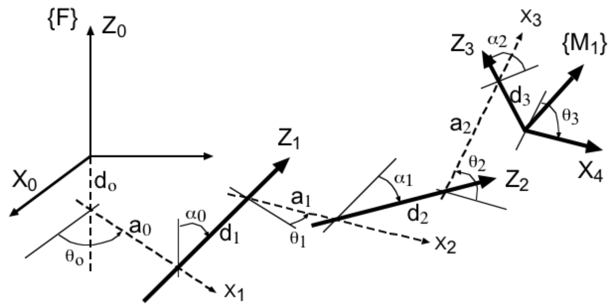


Literature Review

- **Loop closure equations** along the chain from a reference configuration to each goal configuration.



- **Robot kinematics equations** define the set of positions reachable by the end-effector. Equate to each task position to obtain design equations

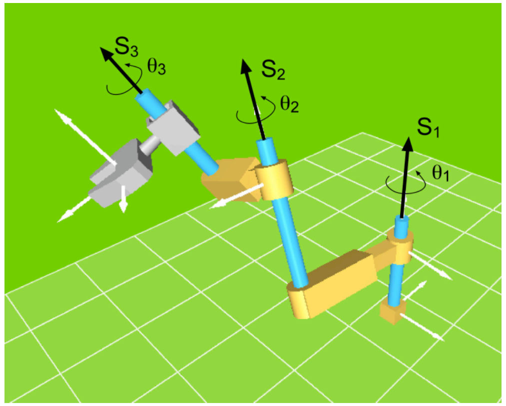


$$[D] = [G][Z(\theta_1, d_1)][X(\alpha_1, a_1)]\ldots[Z(\theta_k, d_k)][H]$$

Literature Review

- **Relative kinematics equations using Clifford algebra** are used to obtain a formulation more directly related to the geometry of the problem.

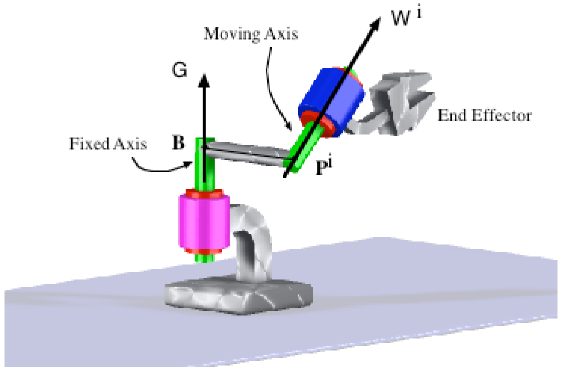
Dual quaternion synthesis is a combination of Kinematic Geometry and Robot Kinematics Equations. It is, in addition, an extension of the complex number formulation to spatial robots.



Challenges of the Synthesis Problem

1. Stating the design equations

- Methods based on geometric constraints give simpler equations but lack a general methodology to find the constraints for all kinds of chains.
- Methods based on the kinematics equations are general but give a more complicated set of equations with extra variables.



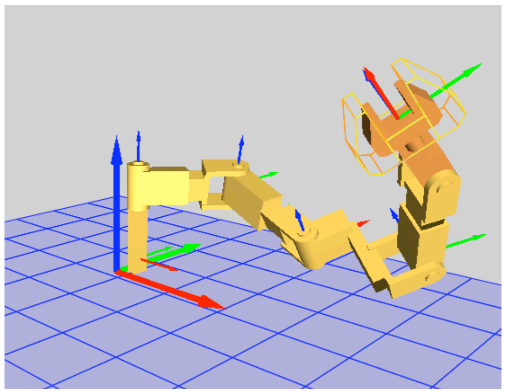
RR chain:
• 10 geometric constraints

$$\mathbf{G}\cdot[A_{1i}-I]\mathbf{V}^1+\mathbf{W}^1\cdot[A_{1i}-I]^T\mathbf{R}+\mathbf{G}\cdot[D_{1i}A_{1i}]\mathbf{W}^1=0,$$

$$\mathbf{G}\cdot[A_{1i}-I]\mathbf{W}^1=0,\quad i=2,3,$$

$$\mathbf{G}\cdot([T_{1i}]\mathbf{P}^1-\mathbf{B})=0,\quad \mathbf{W}^1\cdot(\mathbf{P}^1-[T_{1i}]^{-1}\mathbf{B})=0,$$

$$i=1,2,3.$$

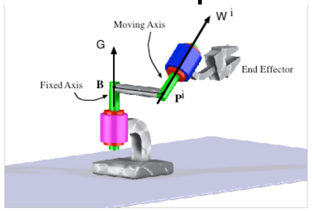


5R chain:
• geometric constraints? *(30 equations)*
• Using the kinematics equations, we obtain a set of 130 equations in 130 variables, including the joint angles.

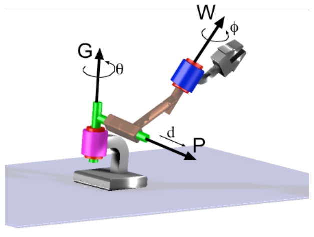
Challenges of the Synthesis Problem

2. Solving the design equations

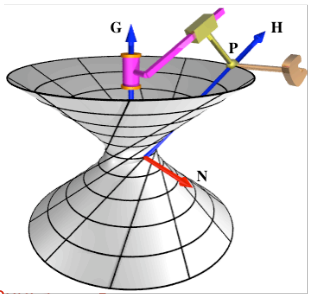
- Set of polynomial equations have a very high total degree.
- The joint variables may be eliminated to reduce the dimension of the problem (But then you increase the degree).
- Due to internal structure, the equations have far less solutions than the Bezout bound (Most likely).
- Some sample cases:



RR chain (2 dof robot):
- Initial total degree: $2^{10} = 1024$.
- Final solution: six roots, with only two real solutions.



RPR chain (3 dof robot):
- Initial total degree: $2^3 * 4^6 = 32768$.
- Final solution: 12 roots. (Algebraic elimination, *Perez and McCarthy, Mechanism and Machine Theory 2005)*

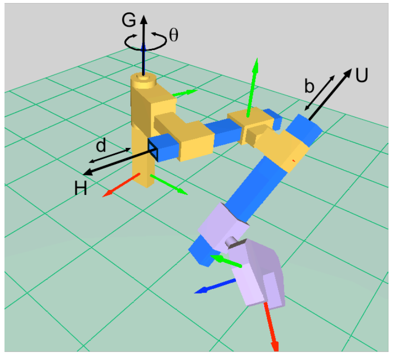


RPS chain (5 dof robot):
- Initial total degree: 262144.
- Final solution: 1024 roots? (Polynomial continuation, *Su and McCarthy, Mechanism and Machine Theory, 2005)*

State of the Art: Solved Synthesis Problems

The following serial chains have been solved:

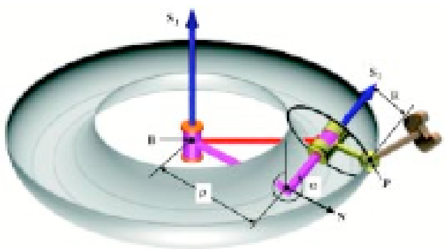
- **Planar and spherical linkages** (early results).
- **Spatial dyads** (2 joints), any type of joint *(Roth, Chen, Tsai, 60´s and 70´s; Innocenti, 90´s).*
- **Spatial 3-jointed chains:**
  - With 2 or less revolute joints: algebraic solutions
    - RPP
    - PRR (*Perez and McCarthy, Mechanism and Machine Theory 2005)*
    - RPC *(Perez and McCarthy, ICAR 2003)*

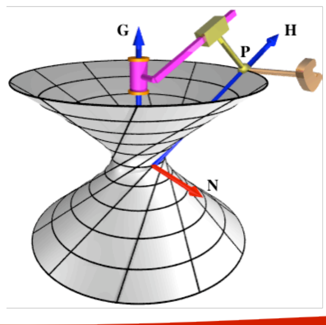


State of the Art (II)

The following serial chains have been solved:

- **Spatial 3-jointed chains**:
  - RRR chain: numerical methods *(Mavroidis)*
    - *Simplified cases, several methods (Lee and Mavroidis 2002, 2004, 2006)*
    - *Interval analysis: 13 real solutions found (Lee, Mavroidis & Merlet, Journal of Mechanical Design, 2004)*
  - XXS chains (with a final spherical joint): “reachable surfaces” (point synthesis only)
    - RPS, simplified solution: algebraic elimination, up to 36 real solutions (*Su and McCarthy, Mechanism and Machine Theory, 2005)*.
    - Polynomial continuation, up to ~100000 roots *(Su, McCarthy & Watson, ASME Journal of Information and Computer Sciences in Engineering, 2004)*





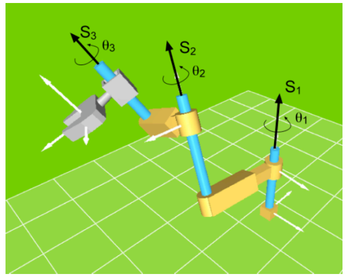
(a)

Synthesis of Robots Using Clifford Algebras

Synthesis of Robots - Outline

1. **Create relative kinematics equations**
   - Composition of relative screw displacements from a reference position using Clifford product.
2. **Counting**
   - Compute n_{max}, maximum number of complete task positions for a desired topology.
3. **Create design equations**
   - Equate dual quaternion kinematics equations to n_{max} task displacements.
4. **Solve the design equations**
   - Solve numerically in parameterized form (including joint variables), or
   - Eliminate the joint variables to obtain a set of reduced equations
     - For those cases where it is possible, algebraic elimination leads to a closed solution:
       - Different algebraic methods (resultant, matrix eigenvalue, …) to create a univariate polynomial for finding all possible solutions.
     - For those cases that are too big for algebraic elimination, numerical methods to find solutions:
       - Polynomial continuation methods.
       - Newton-Raphson numerical methods.

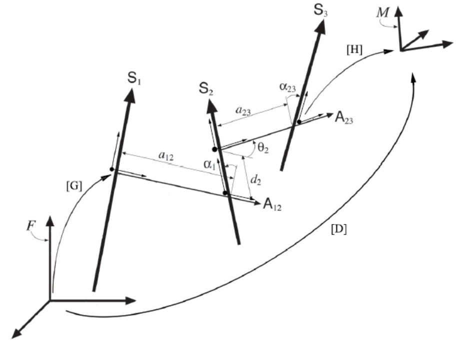
Robot Kinematics Equations



The **kinematics equations** of the robot relate the motion of the end-effector to the composition of transformations about each joint axis.

- Matrix representation using Denavit-Hartenberg parameters:

$$[D_i] = [G][Z(\theta_1^i, d_1^i)][X(\alpha_{12}, a_{12})][Z(\theta_2^i, d_2^i)]$$
$$[X(\alpha_{n-1,n}, a_{n-1,n})][Z(\theta_m^i, d_m^i)][H], \quad i = 1, \ldots n$$



- Relative displacements are used to represent the motion from a reference configuration $[D_0]$,

$$[D_{0i}] = [D_i][D_0]^{-1} = [T(\Delta\theta_1^i, \mathsf{S}_1)][T(\Delta\theta_2^i, \mathsf{S}_2)] \ldots [T(\Delta\theta_m^i, \mathsf{S}_m)]$$

- The transformation can be written as the exponential of a Lie algebra element:

$$[T(\Delta\theta_i, \Delta d_i, \mathsf{S}_i)] = e^{\Delta\theta_i J_i}$$

- The relative kinematics equations, expressed as a product of exponentials, take the form:

$$[D(\Delta\vec{\theta})] = e^{\Delta\theta_1 J_1} e^{\Delta\theta_2 J_2} \ldots e^{\Delta\theta_m J_m}$$

Clifford Algebra Formulation

The Clifford algebra (W.K. Clifford, 1876):

- An algebra with an underlying vector space and a Clifford, or geometric, product to multiply its elements.

- This product can be seen as composed of an inner product (bilinear form) and an outer product (wedge product).

$$ab = \langle a, b \rangle + a \wedge b$$

- Geometric algebras: Clifford algebras in which the bilinear form is non-degenerate.

- The elements of the Clifford algebra are linear combinations of multivectors (the same ones that appear in Grassman algebras).

$$A = a_0 + a_1e_1 + a_2e_2 + a_3e_3 + a_4e_4 + a_5e_1e_2 + a_6e_1e_3 + a_7e_2e_3 + a_8e_4e_1 + a_9e_4e_2 + \\ + a_{10}e_4e_3 + a_{11}e_1e_2e_3 + a_{12}e_1e_2e_4 + a_{13}e_1e_3e_4 + a_{14}e_2e_3e_4 + a_{15}e_1e_2e_3e_4$$

$$Q = q_0 + q_1e_2e_3 + q_2e_3e_1 + q_3e_1e_2 + q_4e_4e_1 + q_5e_4e_2 + q_6e_4e_3 + q_7e_1e_2e_3e_4$$

- Clifford algebras are used to model many physical phenomena (depending on the underlying vector space and the bilinear form used).

Clifford Algebra Formulation

- The Clifford algebra of P^3, $C(P^3)$: sixteen-dimensional vector space with a Clifford product that contains a degenerate bilinear form.

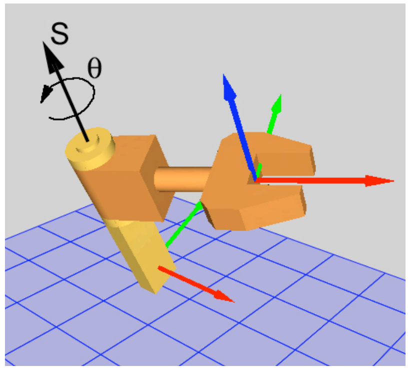
$$< \mathbf{a}, \mathbf{b} >= \mathbf{a}^T \begin{bmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 0 \end{bmatrix} \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3.$$

- Subalgebra of even elements, $C^+(P^3)$: eight-dimensional algebra.

- A spatial displacement is identified with a unit element of the subalgebra (unit dual quaternion).

- For a rotation of angle θ and a slide d about and along the line of Plucker coordinates $\mathsf{S} = \mathbf{S} + \varepsilon \mathbf{S}^0 = \mathbf{S} + \varepsilon \mathbf{C} \times \mathbf{S}$, we can obtain the unit element as the Clifford algebra exponential of the screw axis S:

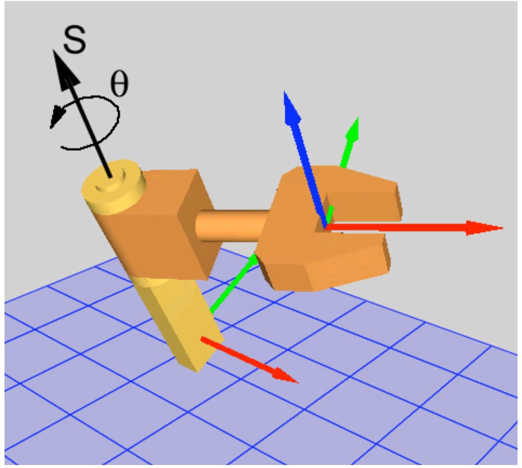
$$\hat{D}(\hat{\theta}) = e^{\frac{\hat{\theta}}{2}\mathsf{S}} = \cos\frac{\hat{\theta}}{2} + \sin\frac{\hat{\theta}}{2}\mathsf{S} = \begin{Bmatrix} \sin\frac{\theta}{2} s_1 \\ \sin\frac{\theta}{2} s_2 \\ \sin\frac{\theta}{2} s_3 \\ \cos\frac{\theta}{2} \end{Bmatrix} + \epsilon \begin{Bmatrix} \sin\frac{\theta}{2} s_1^0 + \frac{d}{2}\cos\frac{\theta}{2} s_1 \\ \sin\frac{\theta}{2} s_2^0 + \frac{d}{2}\cos\frac{\theta}{2} s_2 \\ \sin\frac{\theta}{2} s_3^0 + \frac{d}{2}\cos\frac{\theta}{2} s_3 \\ -\frac{d}{2}\sin\frac{\theta}{2} \end{Bmatrix}$$



Unit Dual Quaternions

Geometric Meaning

In particular, expression of the rotation about a revolute joint of axis S and rotation angle θ:



- **Matrix formulation** (Lie algebra exponential):

$$e^{\theta J} = \begin{bmatrix} e^{\theta \mathbf{S}} & (I - e^{\theta \mathbf{S}})(\mathbf{s} \times (\mathbf{s} \times \mathbf{c})) \\ 0 & 1 \end{bmatrix}$$

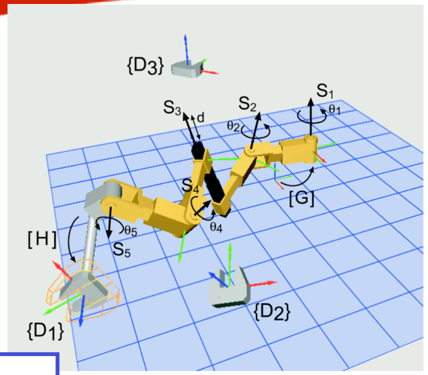
expands to:

$$e^{\theta J} = \begin{bmatrix} 1-(1-c\theta)(s_2^2+s_3^2) & s_1 s_2(1-c\theta) - s_3 s\theta & s_1 s_3(1-c\theta)+s_2 s\theta & c_1(1-c\theta)+(c_2 s_3 - c_3 s_2)s\theta \\ s_1 s_2(1-c\theta)+s_3 s\theta & 1-(1-c\theta)(s_1^2+s_3^2) & s_2 s_3(1-c\theta)-s_1 s\theta & c_2(1-c\theta)+(c_3 s_1 - c_1 s_3)s\theta \\ s_1 s_3(1-c\theta)-s_2 s\theta & s_2 s_3(1-c\theta)+s_1 s\theta & 1-(1-c\theta)(s_1^2+s_2^2) & c_3(1-c\theta)+(c_1 s_2 - c_2 s_1)s\theta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Dual quaternion formulation** (Clifford algebra exponential):

$$e^{\frac{\theta}{2} J} = \begin{Bmatrix} \sin\frac{\theta}{2}(s_1 + \epsilon(c_2 s_3 - c_3 s_2)) \\ \sin\frac{\theta}{2}(s_2 + \epsilon(c_3 s_1 - c_1 s_3)) \\ \sin\frac{\theta}{2}(s_3 + \epsilon(c_1 s_2 - c_2 s_1)) \\ \cos\frac{\theta}{2} \end{Bmatrix}$$

Design Equations using Dual Quaternions



- Specify the m goal positions as relative transformations from the first position P_{1j}, j=2,..,m.
- Create relative kinematics equations for the desired chain,

$$\hat{D}(\Delta\hat{\vec{\theta}}) = e^{\frac{\Delta\hat{\theta}_1}{2}\mathsf{S}_1} e^{\frac{\Delta\hat{\theta}_2}{2}\mathsf{S}_2} \cdots e^{\frac{\Delta\hat{\theta}_n}{2}\mathsf{S}_n}, \\ = (\mathrm{c}\frac{\Delta\hat{\theta}_1}{2} + \mathrm{s}\frac{\Delta\hat{\theta}_1}{2}\mathsf{S}_1)(\mathrm{c}\frac{\Delta\hat{\theta}_2}{2} + \mathrm{s}\frac{\Delta\hat{\theta}_2}{2}\mathsf{S}_2) \cdots (\mathrm{c}\frac{\Delta\hat{\theta}_n}{2} + \mathrm{s}\frac{\Delta\hat{\theta}_n}{2}\mathsf{S}_n).$$

- Create the **design equations**: equate the kinematics equations to each task position written in dual quaternion form:

$$\hat{S}_1(\Delta\hat{\theta}_1^i)\hat{S}_2(\Delta\hat{\theta}_2^i)\ldots\hat{S}_m(\Delta\hat{\theta}_m^i) - \hat{P}_{1i} = 0, \quad i = 2, \ldots, n$$

- We obtain a set of 8-dim. vector equations where the variables to solve for are the **Plucker coordinates of the axes** S_j in the reference position.
- The equations are parameterized by the joint variables θ_j, j=1,…m.

Counting

- How many complete **task positions** can we define?

  Consider a serial chain with r revolute joints and t prismatic joints, and n task positions.

Parameters:

- R joint-- 6 components of a dual vector, $6r$.
- P joint-- 3 components of a direction vector, $3t$.
- Joint variables, $(r+t)(n-1)$, measured relative to initial configuration.

Dual Quaternion design equations, $6(n-1)$ independent equations.

Associated constraint equations:

- R joint-- 2 constraints (Plucker conditions), $2r$.
- P joint-- 1 constraint (unit vector), t .

Imposed extra constraint equations, c.

Equations: $6(n-1)+2r+t+c$. Unknowns: $6r+3t+(r+t)(n-1)$.

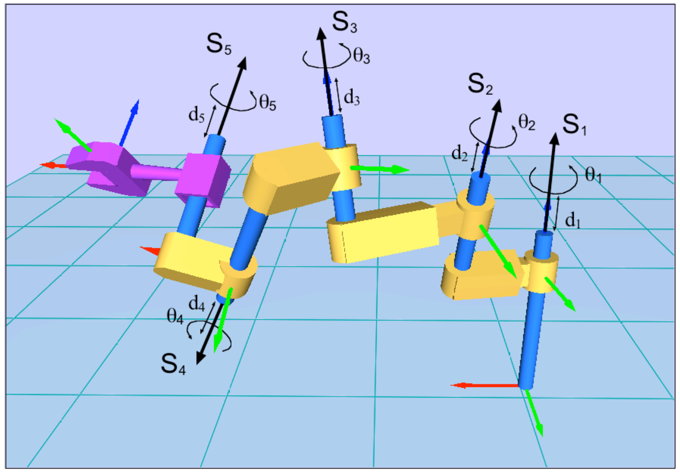
$$n_{max} = (6 + 3r + t - c)/(6 - r - t)$$

(note $r+t < 6$ for constrained robotic systems)

Structure of the Synthesis Equations

Consider the 5C Serial Chain

- 5 cylindrical joints, 10 degrees of freedom.
- Relative kinematics equations for the 5C chain, Clifford algebra exponential form:



$$\hat{Q}_{5C}(\vec{\theta}) = e^{\frac{\hat{\theta}_1}{2}\mathsf{S}_1} e^{\frac{\hat{\theta}_2}{2}\mathsf{S}_2} e^{\frac{\hat{\theta}_3}{2}\mathsf{S}_3} e^{\frac{\hat{\theta}_4}{2}\mathsf{S}_4} e^{\frac{\hat{\theta}_5}{2}\mathsf{S}_5}$$

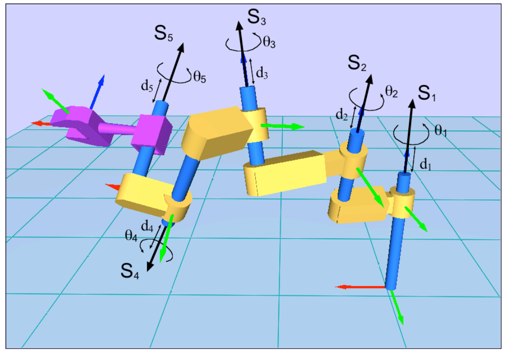
- where each axis is defined as:

$$e^{\frac{\hat{\theta}_i}{2}\mathsf{S}_i} = \begin{Bmatrix} \sin\frac{\theta_i}{2}\mathbf{S}_i \\ \cos\frac{\theta_i}{2} \end{Bmatrix} + \epsilon \begin{Bmatrix} \sin\frac{\theta_i}{2}\mathbf{S}_i^0 + \frac{d_i}{2}\cos\frac{\theta_i}{2}\mathbf{S}_i \\ -\frac{d_i}{2}\sin\frac{\theta_i}{2} \end{Bmatrix}$$

Any 5-jointed serial chain can be derived by specializing the 5C expression.

Structure of the Synthesis Equations

- The equations can be collected as terms in the products of joint variables,
  - Products of the sines and cosines of the joint angles (32 monomials),



$$\mathbf{V} = \{s_1 s_2 s_3 s_4 s_5, (s_1 s_2 s_3 s_4 c_5)_5, (s_1 s_2 s_3 c_4 c_5)_{10}, (s_1 s_2 c_3 c_4 c_5)_{10}, (s_1 c_2 c_3 c_4 c_5)_5, c_1 c_2 c_3 c_4 c_5\}$$

  - plus the terms containing the slides. Total: 192 monomials,

$$\mathbf{M} = \{\mathbf{V}, \frac{\Delta d_1}{2}\mathbf{V}, \frac{\Delta d_2}{2}\mathbf{V}, \frac{\Delta d_3}{2}\mathbf{V}, \frac{\Delta d_4}{2}\mathbf{V}, \frac{\Delta d_5}{2}\mathbf{V}\}$$

- Write the kinematics equations as the sum of terms,

$$\hat{Q}_{5C} = \sum_{i=1}^{192} \mathbf{K}_i M_i, \quad M_i \in \mathbf{M},$$

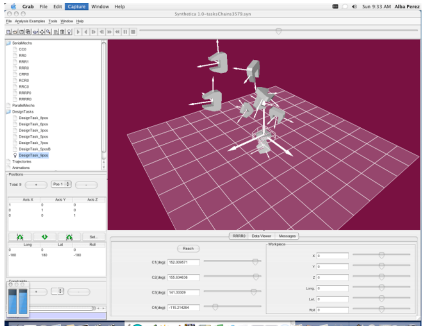
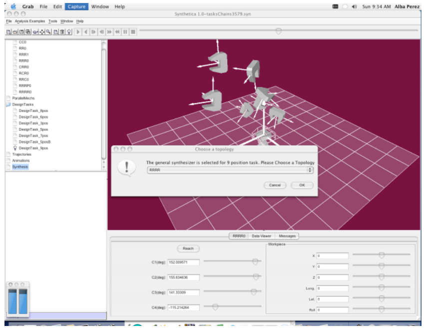
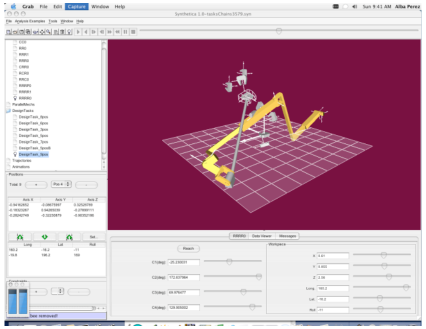
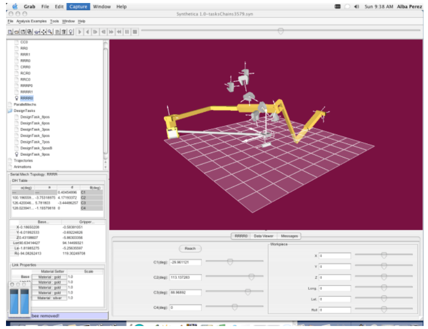
where the 8-dimensional column vectors $\mathbf{K}_i$ contain the structural variables of the joint axes.

Joints	2	3	4	5
Terms	12	32	80	192

Synthetica 3.0: General Synthesizer

- This synthesis method has been implemented in Synthetica (java-based program for visualization, analysis and synthesis of spatial robots).

- A total of 120 topologies consisting of R, P, C, T and S joints and ranging from 2 to 5 degrees of freedom.

- Design equations are generated automatically based on the topology of the chain and equated to the task positions.

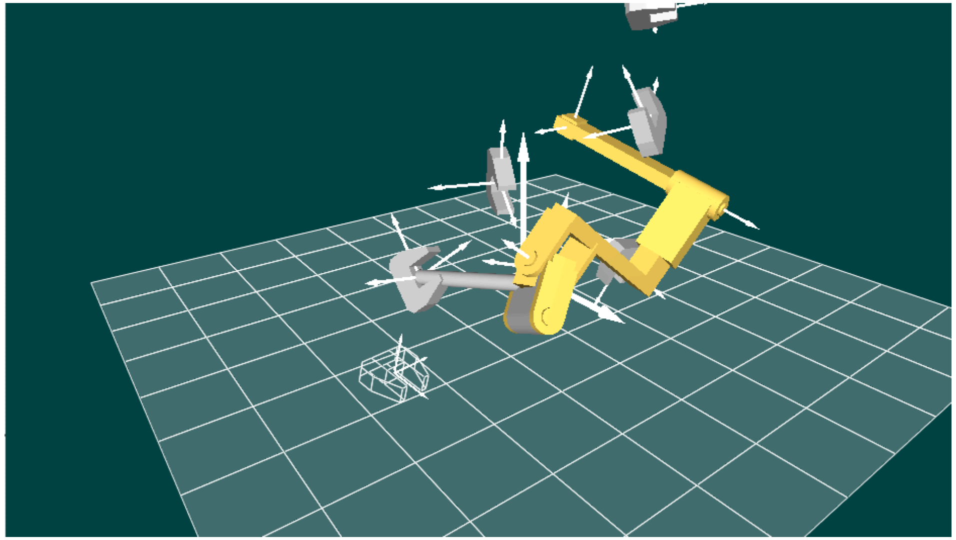
- The counting formula allows to assign serial chain topologies to each task.

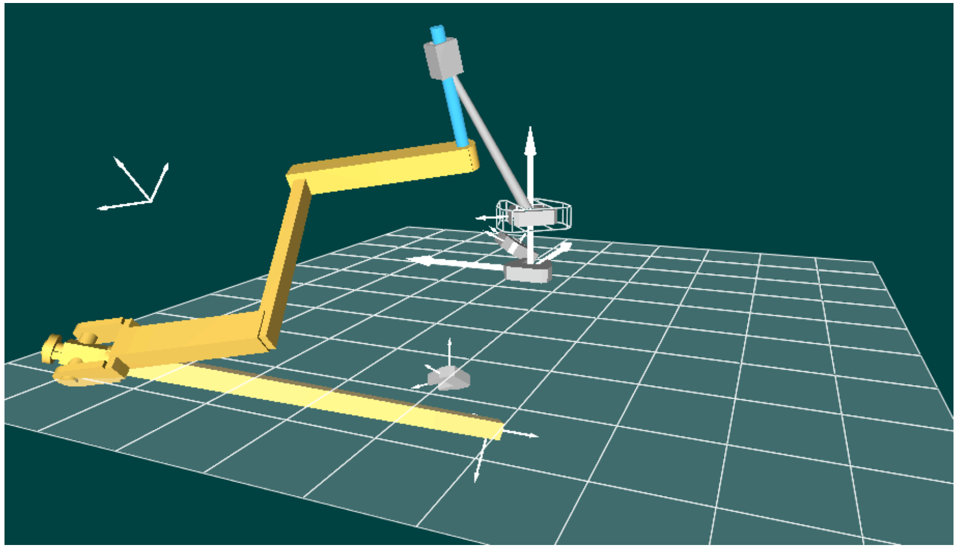
- The solver is a Java translation of FORTRAN Minpack numerical solver (Steve Verrill, translator).

Synthetica 3.0: General Synthesizer

Some examples:

- RR, 3 positions: 83msec
- RRR, 5 positions: 1.55sec
- TC, 6 positions: 245 msec
- TRP, 7 positions: 4.73sec
- RCC, 7 positions: 2.26sec
- RPRR, 8 positions: 2.69sec
- SC, 8 positions: 6.76sec
- RTR, 8 positions: 5.20sec
- RRRR, 9 positions: 6.35sec.
- SPR, 10 positions: 49.72sec.





Open Problems in Kinematic Synthesis

- Algebraic solution for simple chains (RRR chain).
- Synthesis of serial chains with more than 3 revolute joints.
- Computational synthesis.
- Kinematic synthesis of parallel robots.
- Kinematic synthesis in projective environments.
- Synthesis of lattice mechanisms.

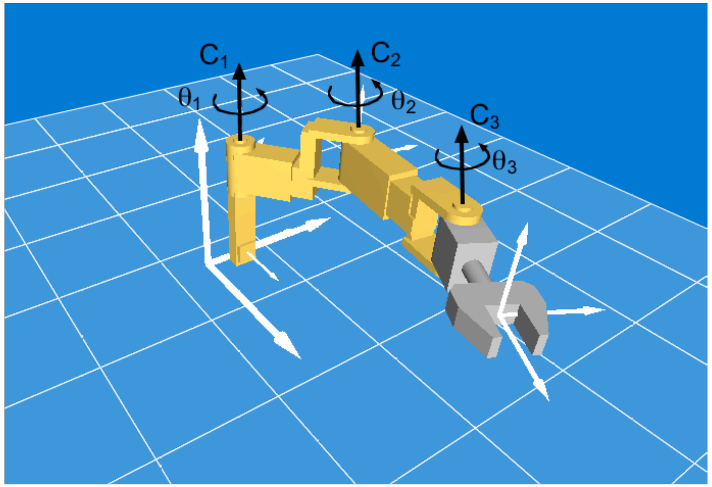
Open Problems in Kinematic Synthesis

1. Algebraic solution for the RRR chain

Interval analysis, homotopy continuation and other numerical results seem to indicate that the problem could be small enough to be solved using algebraic methods.

Numerical results: 13 real solutions (5 days, interval analysis) - Mavroidis

10, 12 or 18 real solutions (~7 hours, 10000 runs) - me



Needed: good design equations.

Open Problems in Kinematic Synthesis

2. Are open chains with more than 3 dof solvable (in a traditional way)?

Is it really a "complexity in the middle" problem?

For most of these chains, we can only state parameterized equations.

- **?** Solutions of the inverse kinematics add to the set of roots and increase the total degree.
- **?** "Simple" algebraic equations exist, if we could state them or simplify the parameterized equations.

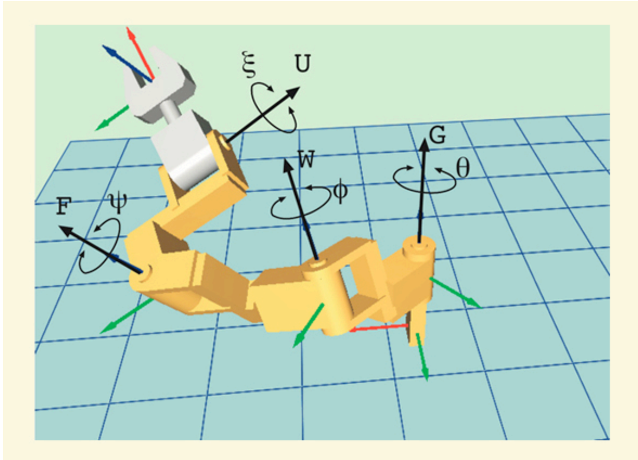
Needed:

- **A study on the set of solutions for 4 and 5 dof robots**
- **An understanding of the effect of subgroups of SE(3) on the synthesis problem.**

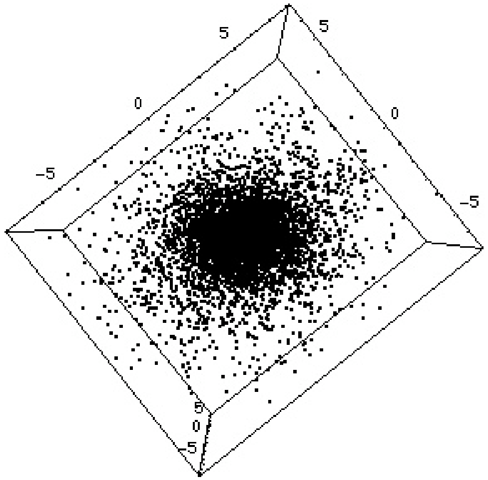
Open Problems in Kinematic Synthesis

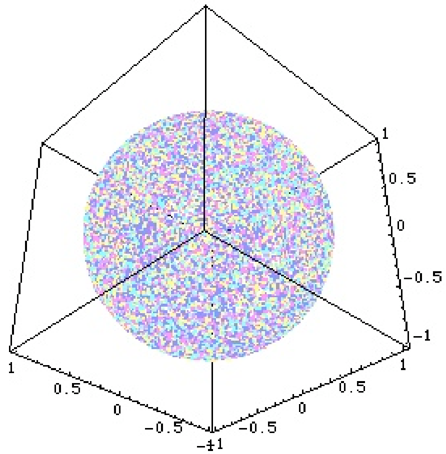
Numerical results: A very high number of solutions:

RR:	**2 real solutions**
RRR:	**approx. 18 real solutions**
RRRR:	**thousands?**
RRRRR:	**many thousands?**



Total	Total non-repeated	% different
172	167	97,09
403	381	94,54
486	456	93,83
688	642	93,31
5028	4366	86,83
5962	5123	85,93
6889	5468	79,37
10000	6534	65,34
20000	14963	74,815





The strategy for the synthesis will be different if almost everything is a solution.

Open Problems in Kinematic Synthesis

3. Computational synthesis: Create a general, robust solver for any serial chain.

Synthetica 3.0 has a general solver based on Clifford algebra kinematics equations that uses a Levenberg-Marquardt algorithm to obtain one solution at a time.

- Java-based.
- Robust.
- Slow (in creating the kinematics equations).
- Does not handle particular cases.
- One (real) solution for each run.

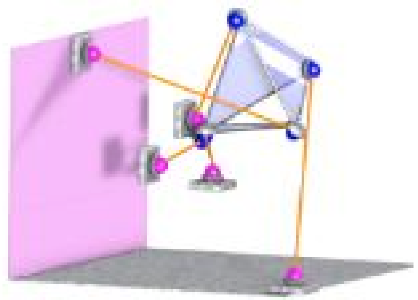
Needed: A faster solver that can handle specific constraints between axes and, if possible, provides with all solutions.

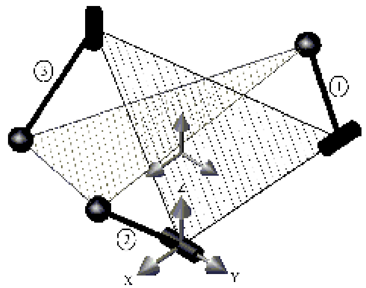
Open Problems in Kinematic Synthesis

4. Kinematic synthesis of parallel robots

Can we apply the synthesis theory to parallel robots?

- **Traditional way:** solve the kinematic synthesis for serial robots and join together several of the solutions.
  - Liao and McCarthy, "On the Seven Position Synthesis of a 5-SS Platform Linkage," ASME Journal of Mechanical Design, 2001.
  - Kim and Tsai, "Kinematic Synthesis of Spatial 3-RPS Parallel Manipulators," *Proc. ASME DETC 2002*.





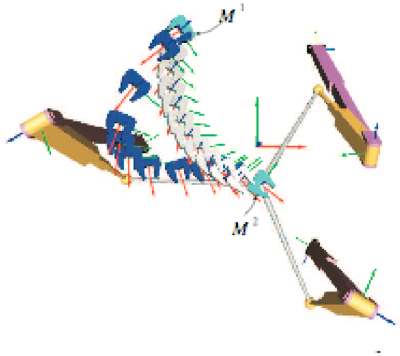
- Problems:
  - We cannot guarantee that the parallel robot can move from one position to the next.
  - We cannot impose geometric constraints among legs.

Open Problems in Kinematic Synthesis

4. Kinematic synthesis of parallel robots

Can we apply the synthesis theory to parallel robots?

- **The other way:** state and solve the equations of all legs together
  - Wolbrecht, Su, Perez and McCarthy, "Geometric Design Of Symmetric 3-RRS Constrained Parallel Platforms," *ASME IMECE*, 2004.



- Problem: the general counting misses the specific relations between legs.

Needed:

- **A lot of work.**
- **A counting scheme that includes information about the workspace of the parallel robot?**

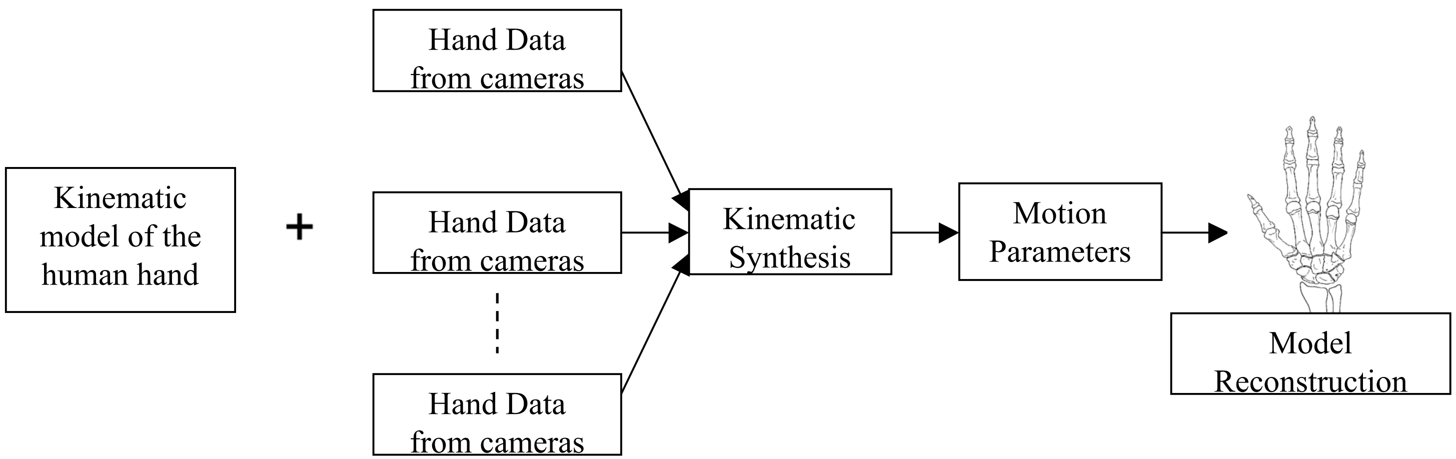
Open Problems in Kinematic Synthesis

5. Kinematic synthesis in projective environments

Can we extract skeleton information from visual data?

Given a set of images of a moving kinematic chain, we want to know the type and position of joints and the joint variables for performing the motion.

- Diagnosis, rehabilitation, system identification.
- Compact description of motion.

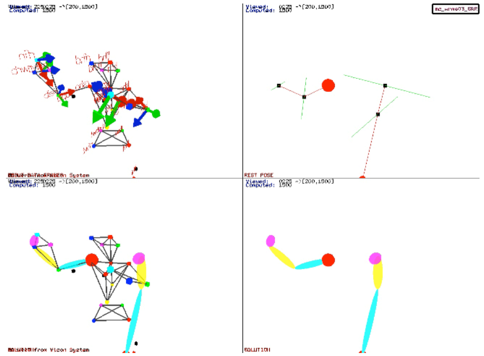


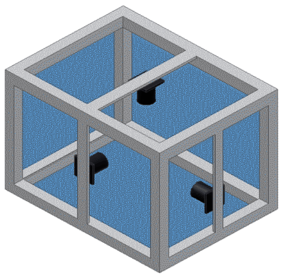
5. Kinematic synthesis in projective environments

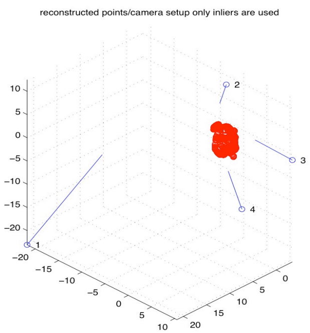
Two approaches:

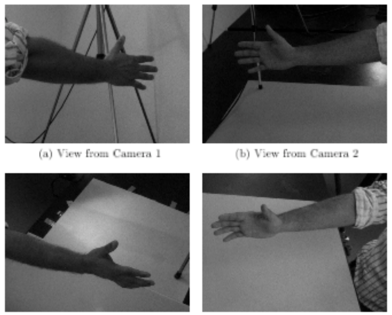
a) Use several cameras to reconstruct a 3D image, then apply kinematic synthesis on the Euclidean space.

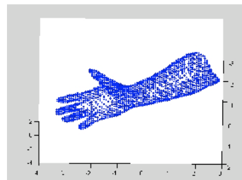
- Villa-Uriol et al.,“Kinematic Synthesis of Avatar Skeletons from Visual Data”, *ARK*2004.











Synthesis solver

Synthesis is easy, image processing (3D reconstruction, segmentation, classification and tracking) complicated.

Open Problems in Kinematic Synthesis

5. Kinematic synthesis in projective environments

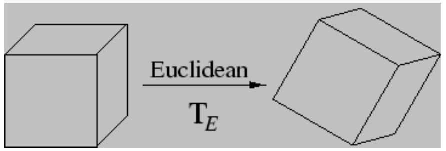
b) Apply kinematic synthesis on each 2D image

- kinematic synthesis needs to be modified to deal with a projective transformation (constant) + several rigid body transformations (different for each frame).

Rigid body motion: 6 parameters, 8-dim algebra.

Projective transformation: 15 parameters, 16-dim algebra.

$$\mathbf{T}_E = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$\mathbf{T}_P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix}$$

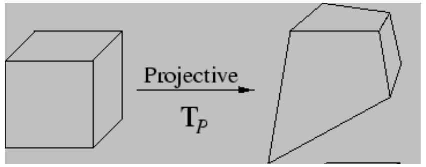
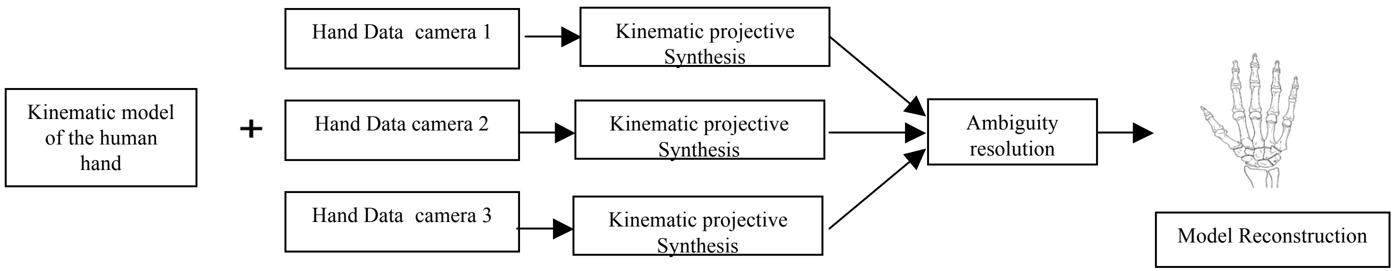


Image processing simpler, synthesis becomes more complicated.



Needed: Kinematic synthesis theory for non-rigid (projective) motion.

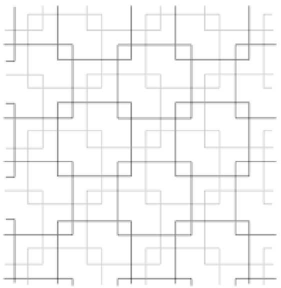
Open Problems in Kinematic Synthesis

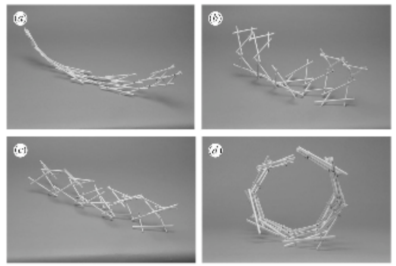
6. Synthesis of lattice mechanisms

Lattice mechanisms: Mechanisms composed of repeated individual cells. Each cell is an (overconstrained) parallel robot.

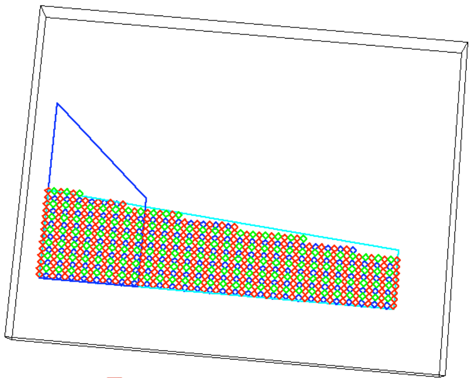
Previous work:

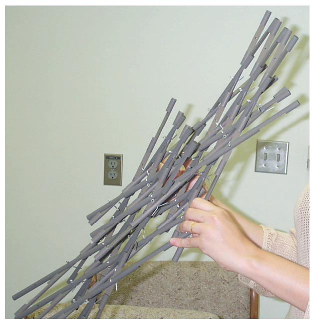
- You & Pellegrino (Oxford, 2003) : parallelograms, planar and spherical, and Chen (2003 -): Bennett-based linkages

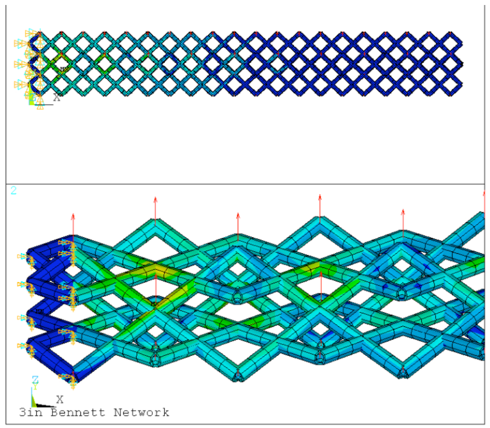




- Us (not published, 2003): Bennett linkages.



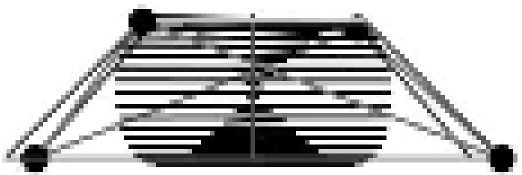


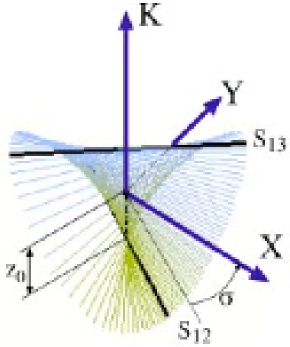


Open Problems in Kinematic Synthesis

6. Synthesis of lattice mechanisms

- Work has been done on the traditional kinematic analysis of the mechanism.
- Design is being done in an “ad hoc” way (no general methodology).
- Overconstrained linkages have symmetries that could be used to simplify the analysis and to state design equations (done for the Bennett linkage only).





Needed:

- **More work on identifying symmetries on the workspace of overconstrained linkages.**
- **Use of these symmetries to state efficient synthesis equations.**
- **A methodology to apply the synthesis along the lattice structure.**

Conclusions

- The finite-position synthesis for general topologies is still an open problem.
- Equating the relative transformation of the chain to the set of desired relative transformations yields a general method to formulate design equations.
- Expressing the displacements using the Clifford algebra of dual quaternions helps in reducing the complexity of the equations.
- For serial chains with four or more joints, complexity may be a real problem (independently of the method used to state the equations).
- There are many interesting problems to work on in kinematic synthesis !

Additional Slides

- References for kinematic synthesis.
- Examples of dual quaternion synthesis.
- Kinematic synthesis of skeleton from visual data: actual methodology and results.

Literature Review

Geometric features of the chain are used to formulate the algebraic constraint equations. (distance and angle constraints)

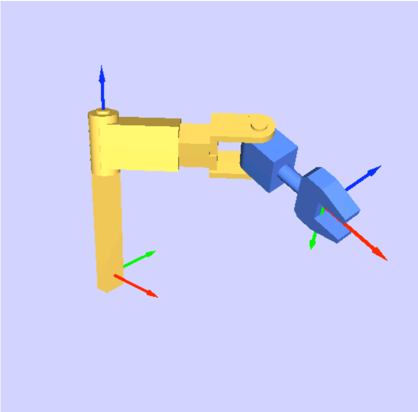
• Roth, B., 1968, "The design of binary cranks with revolute, cylindric, and prismatic joints", *J. Mechanisms*, 3(2):61-72.

• Chen, P., and Roth, B., 1969, ''Design Equations for the Finitely and Infinitesimally Separated Position Synthesis of Binary Links and Combined Link Chains,'' *ASME J. Eng. Ind*. 91(1):209–219.

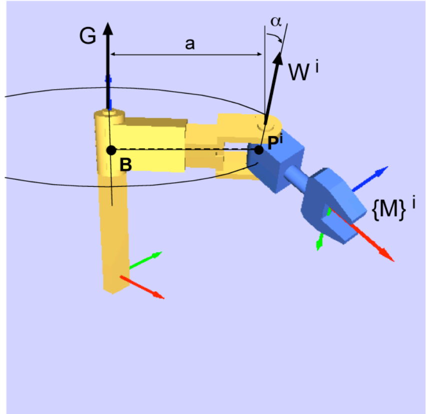
• Innocenti, C., 1994, ``Polynomial Solution of the Spatial Burmester Problem.'' *Mechanism Synthesis and Analysis,* ASME DE vol. 70.

• Nielsen, J. and Roth, B., 1995, "Elimination Methods for Spatial Synthesis," *Computational Kinematics*, (eds. J. P. Merlet and B. Ravani), Vol. 40 of Solid Mechanics and Its Applications, pp. 51-62, Kluwer Academic Publishers.

• Kim, H. S., and Tsai, L. W., 2002, "Kinematic Synthesis of Spatial 3-RPS Parallel Manipulators," *Proc. ASME Des. Eng. Tech. Conf.* paper no. DETC2002/MECH-34302, Sept. 29-Oct. 2, Montreal, Canada.

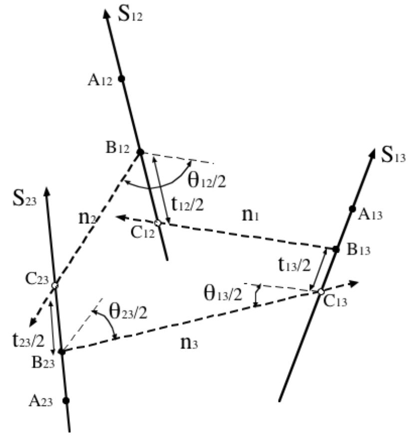


RR chain



Literature Review

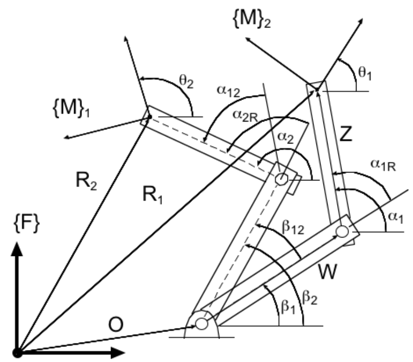
- **Kinematic geometry** based on the screw representation of the composition of displacements. (equivalent screw triangle)



  - Tsai, L. W., and Roth, B., 1972, "Design of Dyads with Helical, Cylindrical, Spherical, Revolute and Prismatic Joints," *Mechanism and Machine Theory*, 7:591-598.
  - Tsai, L.W., and Roth, B., "A Note on the Design of Revolute-Revolute Cranks," *Mechanism and Machine Theory*, Vol. 8, pp. 23-31, 1973.

- **Loop closure equations** along the chain from a reference configuration to each goal configuration.

  - Sandor, G. N., and Erdman, A. G., 1984, *Advanced Mechanism Design: Analysis and Synthesis*, Vol. 2. Prentice-Hall, Englewood Cliffs, NJ
.
  - Sandor, G.N., Xu, Y., and Weng, T.C., 1986, "Synthesis of 7-R Spatial Motion Generators with Prescribed Crank Rotations and Elimination of Branching", *The International Journal of Robotics Research*, 5(2):143-156.
  - Sandor, G.N., Weng, T.C., and Xu, Y., 1988, "The Synthesis of Spatial Motion Generators with Prismatic, Revolute and Cylindric Pairs without Branching Defect", Mechanism and Machine Theory, 23(4):269-274.



Literature Review

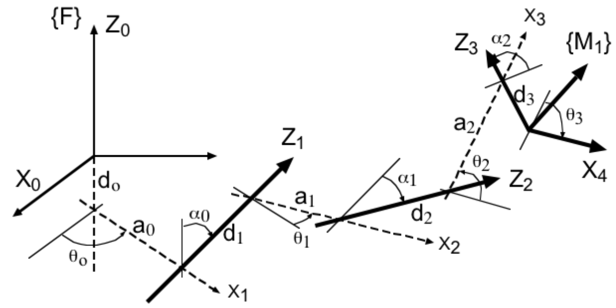
- **Robot kinematics equations** define the set of positions reachable by the end-effector. Equate to each task position to obtain design equations

- Park, F. C., and Bobrow, J. E., 1995, "Geometric Optimization Algorithms for Robot Kinematic Design". *Journal of Robotic Systems*, 12(6):453-463.

- Mavroidis, C., Lee, E., and Alam, M., 2001, A New Polynomial Solution to the Geometric Design Problem of Spatial RR Robot Manipulators Using the Denavit-Hartenberg Parameters, *J. Mechanical Design*, 123(1):58-67.

- Lee, E., and Mavroidis, D., 2002, "Solving the Geometric Design Problem of Spatial 3R Robot Manipulators Using Polynomial Homotopy Continuation", ASME J. of Mechanical Design, 124(4), pp.652-661.

- Lee, E., and Mavroidis, D., 2002c, "Geometric Design of Spatial PRR Manipulators Using Polynomial Elimination Techniques," Proc. ASME 2002 Design Eng. Tech. Conf., paper no. DETC2002/MECH-34314, Sept. 29-Oct. 2, Montreal, Canada.



$$[D] = [G][Z(\theta_1, d_1)][X(\alpha_1, a_1)] \ldots [Z(\theta_k, d_k)][H]$$

Literature Review

• **Relative kinematics equations using Clifford algebra** are used to obtain a formulation more directly related to the geometry of the problem.

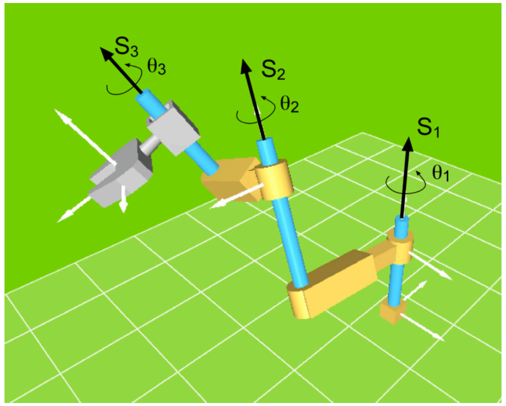
• Perez Gracia, A. and McCarthy, J.M., "The Kinematic Synthesis of Spatial Serial Chains Using Clifford Algebra Exponentials", Proceedings of the Institution of Mechanical Engineers, Part C, Journal of Mechanical Engineering Science, 220(7): 953-968, 2006.

• Perez, A. and McCarthy, J.M., "Clifford Algebra Exponentials and Planar Linkage Synthesis Equations", *ASME Journal of Mechanical Design*, 127(5): 931-940, September 2005.

•Perez, A. and McCarthy, J.M., "Geometric Design of RRP, RPR and PRR Serial Chains", *Mechanism and Machine Theory*, 40(11):1294-1311, November 2005.

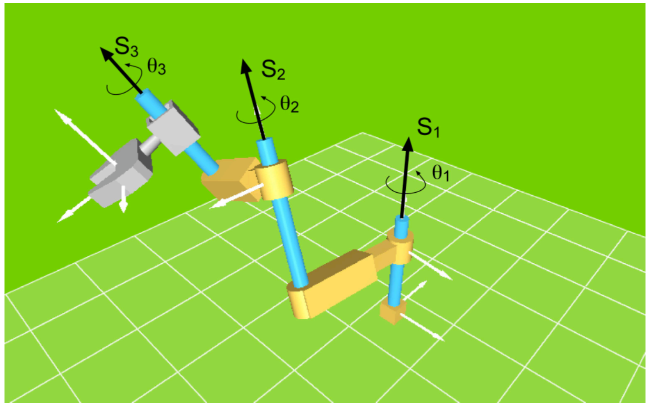
•Perez, A. and McCarthy, J.M., "Sizing a Serial Chain to Fit a Task Trajectory Using Clifford Algebra Exponentials", *2005 IEEE International Conference on Robotics and Automation*, April 18-22, 2005, Barcelona.

•Perez, A. and McCarthy, J.M., "Dual Quaternion Synthesis of Constrained Robotic Systems", *ASME Journal of Mechanical Design*, 126(3): 425-435, 2004.



Example: The 3C Spatial Serial Chain

- Clifford algebra relative kinematics equations,



$$\hat{D}(\Delta\vec{\hat{\theta}}) = (\mathrm{c}\frac{\Delta\hat{\theta}_1}{2} + \mathrm{s}\frac{\Delta\hat{\theta}_1}{2}\mathsf{S}_1)(\mathrm{c}\frac{\Delta\hat{\theta}_2}{2} + \mathrm{s}\frac{\Delta\hat{\theta}_2}{2}\mathsf{S}_2)(\mathrm{c}\frac{\Delta\hat{\theta}_3}{2} + \mathrm{s}\frac{\Delta\hat{\theta}_3}{2}\mathsf{S}_3),$$

- Expand to:

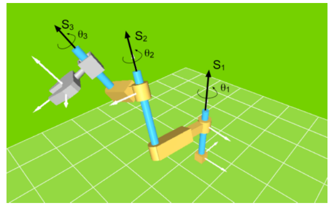
$$\begin{aligned}\hat{D}(\Delta\vec{\hat{\theta}}) =& (\hat{c}_1\hat{c}_2 - \hat{s}_1\hat{s}_2\mathsf{S}_1\cdot\mathsf{S}_2 + \hat{s}_1\hat{c}_2\mathsf{S}_1 + \hat{c}_1\hat{s}_2\mathsf{S}_2 + \hat{s}_1\hat{s}_2\mathsf{S}_1\times\mathsf{S}_2)(\hat{c}_3 + \hat{s}_3\mathsf{S}_3),\\ =& \hat{c}_1\hat{c}_2\hat{c}_3 - \hat{s}_1\hat{s}_2\hat{c}_3\mathsf{S}_1\cdot\mathsf{S}_2 - \hat{s}_1\hat{c}_2\hat{s}_3\mathsf{S}_1\cdot\mathsf{S}_3 - \hat{c}_1\hat{s}_2\hat{s}_3\mathsf{S}_2\cdot\mathsf{S}_3 - \hat{s}_1\hat{s}_2\hat{s}_3\mathsf{S}_1\times\mathsf{S}_2\cdot\mathsf{S}_3\\ &+ \hat{s}_1\hat{c}_2\hat{c}_3\mathsf{S}_1 + \hat{c}_1\hat{s}_2\hat{c}_3\mathsf{S}_2 + \hat{s}_1\hat{s}_2\hat{c}_3\mathsf{S}_1\times\mathsf{S}_2 + \hat{c}_1\hat{c}_2\hat{s}_3\mathsf{S}_3 - \hat{s}_1\hat{s}_2\hat{s}_3(\mathsf{S}_1\cdot\mathsf{S}_2)\,\mathsf{S}_3\\ &+ \hat{s}_1\hat{c}_2\hat{s}_3\mathsf{S}_1\times\mathsf{S}_3 + \hat{c}_1\hat{s}_2\hat{s}_3\mathsf{S}_2\times\mathsf{S}_3 + \hat{s}_1\hat{s}_2\hat{s}_3(\mathsf{S}_1\times\mathsf{S}_2)\times\mathsf{S}_3. \qquad (\end{aligned}$$

- The kinematics equations can be written as a linear combination of the products of joint variables:

$$\mathbf{M} = (\mathbf{V}, \frac{\Delta d_1}{2}\mathbf{V}, \frac{\Delta d_2}{2}\mathbf{V}, \frac{\Delta d_3}{2}\mathbf{V}),$$

$$\hat{\mathbf{V}} = (\hat{c}_1\hat{c}_2\hat{c}_3, \hat{s}_1\hat{c}_2\hat{c}_3, \hat{c}_1\hat{s}_2\hat{c}_3, \hat{s}_1\hat{s}_2\hat{c}_3, \hat{c}_1\hat{c}_2\hat{s}_3, \hat{s}_1\hat{c}_2\hat{s}_3, \hat{c}_1\hat{s}_2\hat{c}_3, \hat{s}_1\hat{s}_2\hat{s}_3)^T.$$

Example: The 3C Spatial Serial Chain



- Finally, we can write the equations as the linear system:

$$\hat{\mathrm{D}}(\Delta\hat{\vec{\theta}}) =$$

$$\begin{bmatrix} 0 & S_1 & S_2 & S_1 \times S_1 & S_3 & S_1 \times S_3 & S_2 \times S_3 & -(S_1 \cdot S_2)\, S_3 + (S_1 \times S_2) \times S_3 \\ 1 & 0 & 0 & -S_1 \cdot S_2 & 0 & -S_1 \cdot S_3 & -S_2 \cdot S_3 & -S_1 \times S_2 \cdot S_3 \end{bmatrix} \hat{\mathrm{V}}.$$

- The equations of the CCC chain can be specialized to those of any 3-jointed robot

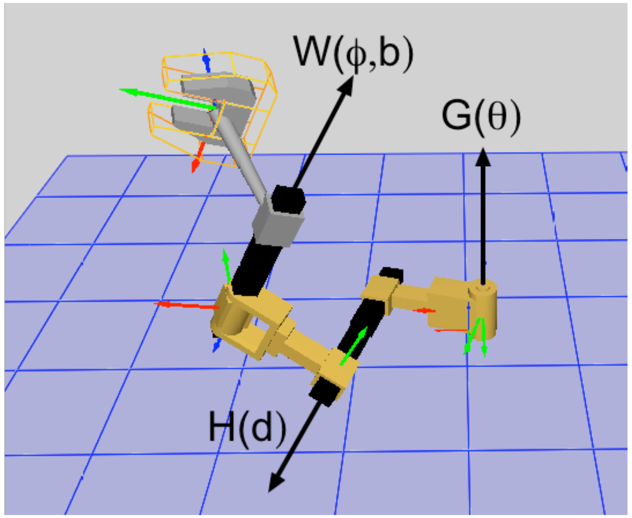
Serial Chain	Condition	Terms
CCR	d3 = 0	24
CRR	d2 = 0, d3 = 0	16
RRR	d1 = d2 = d3 = 0	8
CCP	s3 = 0	16
CPP	s2 = 0, s3 = 0	8
CPR	s2 = 0, d3 = 0	12
RRP	d1 = d2 = 0, s3 = 0	8
RPP	d1 = 0, s2 = s3 = 0	6
CT	d2 = 0, d3 = 0	16
RT	d1 = d2 = d3 = 0	8
PT	s1 =0, d2 = d3 = 0	8

Design Example: RPC Robot

- **Design equations**

$$\hat{Q}_{RPC}(\theta, d, \phi, b) = \hat{G}(\theta, 0)\hat{H}(0, d)\hat{W}(\phi, b)$$

$$\hat{Q}_{RPC}(\theta^i, d^i, \phi^i, b^i) = \hat{P}^i$$

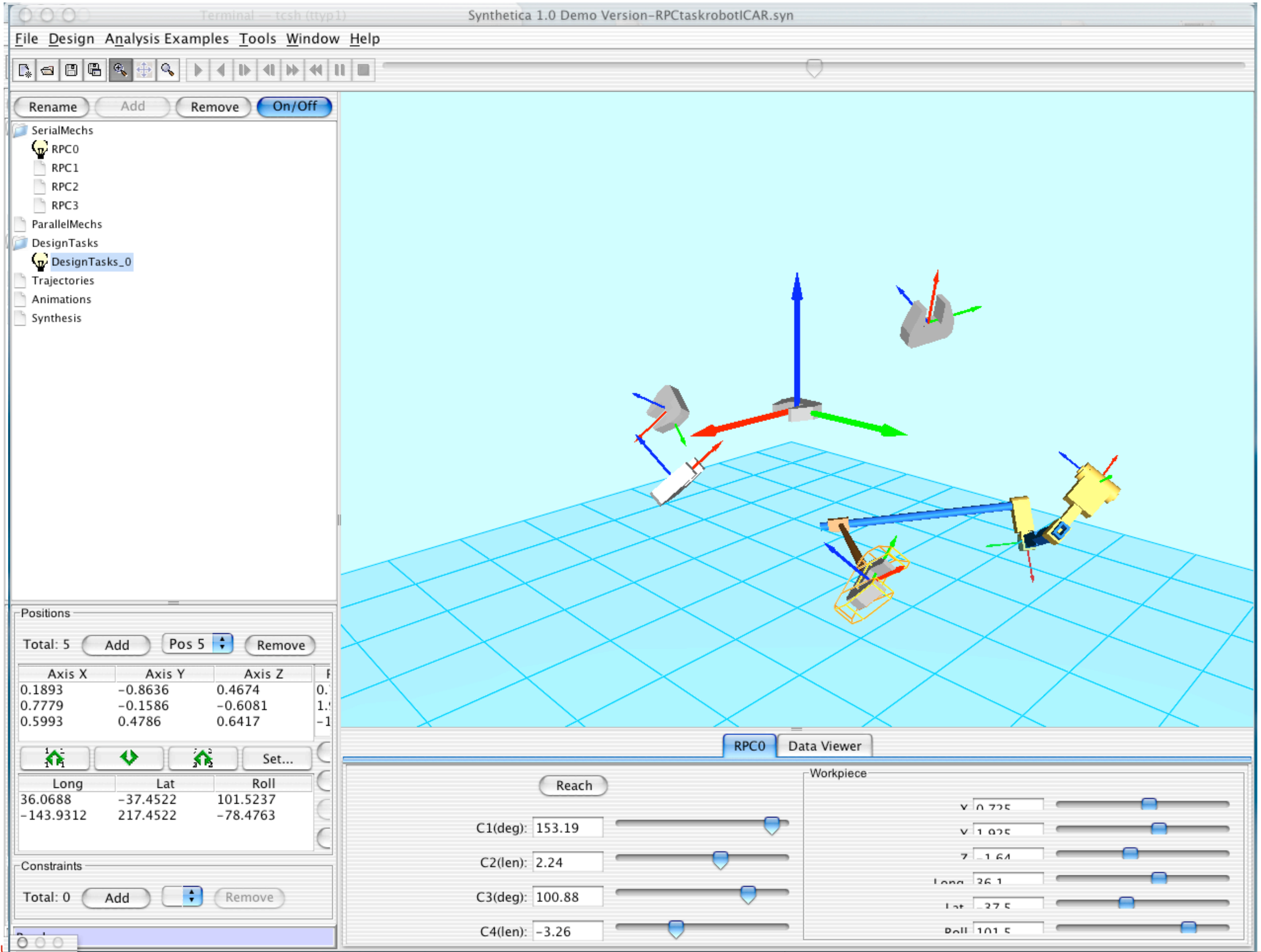


- We can define a **maximum** of n=5 task positions if we impose the conditions g.h=0, w.h=0.
- From the initial set of 34 equations, eliminate the joint variables to obtain a set of 15 equations in 15 parameters,

$$\begin{aligned}
&\{R_1, R_2\}^i, \quad i = 1, \ldots 4,\\
&\mathbf{g}\cdot\mathbf{g} = 1, \quad \mathbf{g}\cdot\mathbf{g}_0 = 0,\\
&\mathbf{w}\cdot\mathbf{w} = 1, \quad \mathbf{w}\cdot\mathbf{w}_0 = 0,\\
&\mathbf{h}\cdot\mathbf{h} = 1, \quad \mathbf{h}\cdot\mathbf{g} = 0, \quad \mathbf{h}\cdot\mathbf{w} = 0.
\end{aligned}$$

For the RPC chain, we can use resultant methods to obtain a sixth-degree univariate polynomial. The maximum number of solutions is six.

Design Example: RPC Robot



Kinematic Synthesis of the Avatar Skeleton

- **Goal:** To obtain a model for the human body that approximates human motion in a compact way.
- **Skeleton topology:** five serial chains with revolute and spherical joints

- **Forward kinematics:**

  - *Head:* S chain

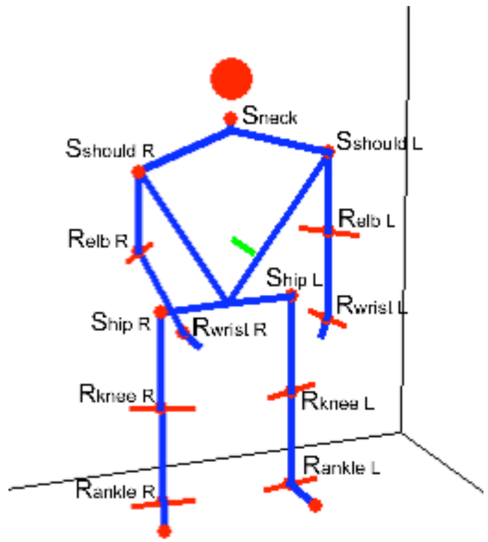
$$\hat{Q}_{head} = \hat{S}_{neck}(\theta_{h1}, \theta_{h2}, \theta_{h3}),$$

  - *Left Arm*
  - *Right Arm:* SRR chains

$$\hat{Q}_{arm} = \hat{S}_{should}(\theta_{a1}, \theta_{a2}, \theta_{a3})\hat{R}_{elb}(\theta_{a4})\hat{R}_{wrist}(\theta_{a5}),$$

  - *Left Leg*
  - *Right Leg:* SRR chains

$$\hat{Q}_{leg} = \hat{S}_{hip}(\theta_{l1}, \theta_{l2}, \theta_{l3})\hat{R}_{knee}(\theta_{l4})\hat{R}_{ankle}(\theta_{l5}).$$



Kinematic Synthesis of the Avatar Skeleton

- **Dual quaternion expression of the kinematics equations**
  - Revolute joint
  - (elbow, wrist, knee, ankle)

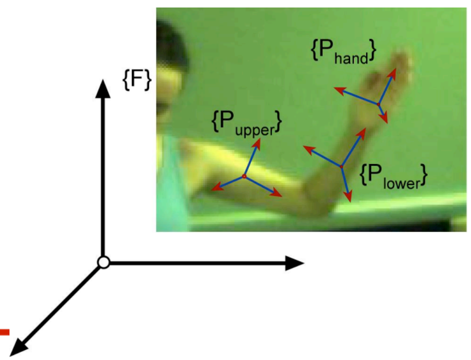
$$\hat{S}(\theta) = \begin{Bmatrix} \sin\frac{\theta}{2}s_x \\ \sin\frac{\theta}{2}s_y \\ \sin\frac{\theta}{2}s_z \\ \cos\frac{\theta}{2} \end{Bmatrix} + \epsilon \begin{Bmatrix} \sin\frac{\theta}{2}s_x^0 \\ \sin\frac{\theta}{2}s_y^0 \\ \sin\frac{\theta}{2}s_z^0 \\ 0 \end{Bmatrix}.$$

  - Spherical joint
  - (neck, shoulder, hip)

$$\hat{S}(\theta_1,\theta_2,\theta_3) = \begin{Bmatrix} \alpha_1\mathbf{g}_1 + \alpha_2\mathbf{g}_2 + \alpha_3\mathbf{g}_3 \\ \alpha_4 \end{Bmatrix} + \epsilon \begin{Bmatrix} \alpha_1\mathbf{g}_1^0 + \alpha_2\mathbf{g}_2^0 + \alpha_3\mathbf{g}_3^0 \\ 0 \end{Bmatrix}$$

- **Limb data**
  - Attaching a moving frame to each limb and for each snapshot, we obtain the set of positions used to dimension the skeleton,

P^j_{upper}, P^j_{lower}, P^j_{hand} for arms and legs, $j=1,..,n$
P^i_{head} for head, $i=1,..,m$.



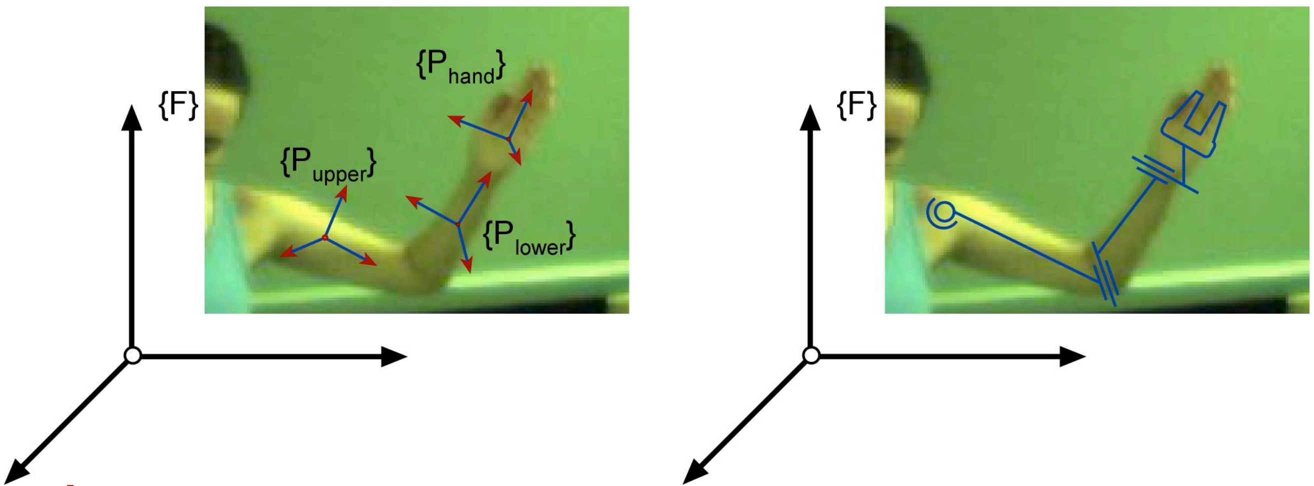
Kinematic Synthesis of the Avatar Skeleton

- **Adjustment of skeleton to limb data**
  - Equate the kinematics equations to experimental limb data

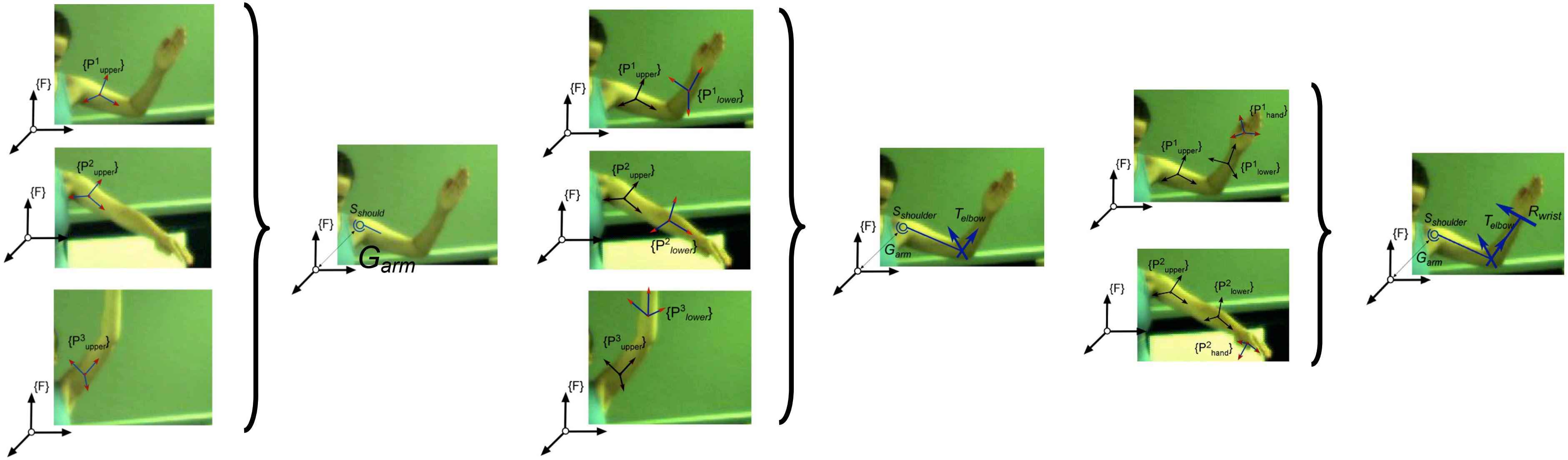
$$\hat{Q}_{head} = \hat{P}^{j}_{head}, \quad j = 2, \ldots n,$$
$$\hat{Q}_{arm} = \hat{P}^{i}_{arm}, \quad \hat{Q}_{leg} = \hat{P}^{i}_{leg}, \quad i = 2, \ldots m,$$

  - Minimize for the joint parameters (location and orientation) and the joint variables

- **Hierarchical solution process**
  - We have information for each link of the skeleton ➡ we can solve sequentially for each joint.



Kinematic Synthesis of the Avatar Skeleton



$$\hat{S}_{shoulder}(\theta^i_{a1},\theta^i_{a2},\theta^i_{a3}) = \hat{G}^{i*}_{arm}\hat{P}^i_{upper}, \quad i=1,\ldots n \qquad \hat{T}_{elbow}(\theta^i_{a4},\theta^i_{a5}) = \hat{P}^{i*}_{upper}\hat{P}^i_{lower}, \quad i=1,\ldots,n \qquad \hat{R}_{wrist}(\theta^i_{a6}) = P^{i*}_{lower}\hat{P}^i_{hand}, \quad i=1,\ldots,n$$

3 input data frames required

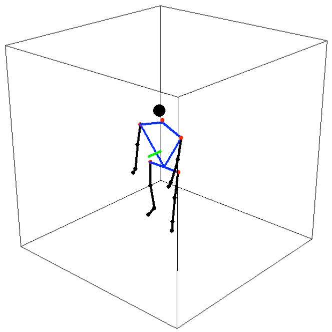
3 input data frames required

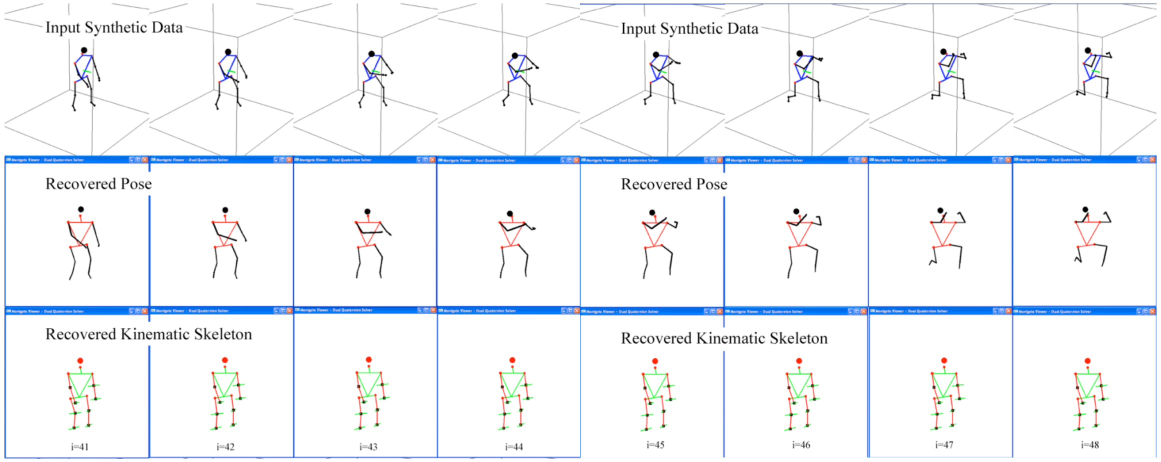
2 input data frames required

Repeat the process for each of the five serial chains. We obtain the Plücker coordinates of the joints and the joint variables to reach the first pose of the subject.

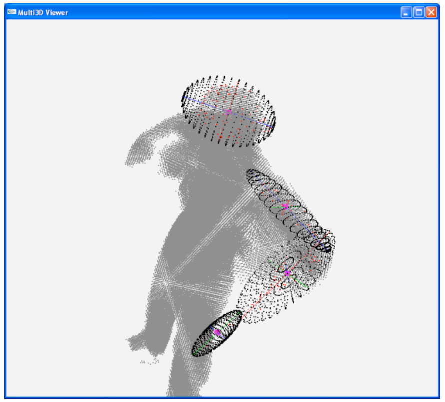
Kinematic Synthesis of the Avatar Skeleton
Experimental Results

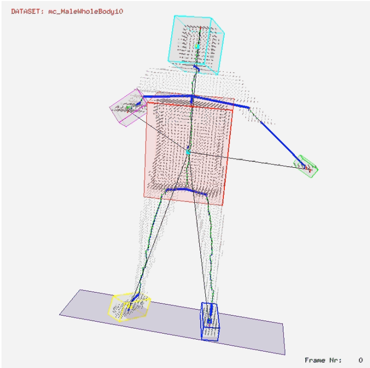
- **Algorithm testing:** synthetically generated frames. 15 frames were needed to obtain a good approximation (error 5e-3)





- **Real video data:** The dataset from raw visual images is very noisy. When motion capture images are used, the results are good.





Kinematic Synthesis of the Avatar Skeleton
Experimental Results

- **Real video data:** The dataset from raw visual images is very noisy. When motion capture images are used, the results are good.

- Several iterations are needed for including all possible joint movements.

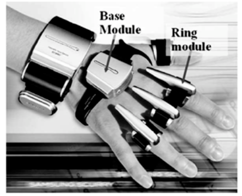
- Once the skeleton is defined, inverse kinematics is trivial. Criteria for termination of synthesis algorithm is needed.

Hand Motion Identification

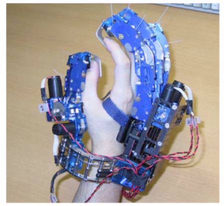
Other Approaches

Glove-based devices
Exoskeleton devices

Ekvall and Kragic, 2005



Kim, Soh and Lee, 2005



Nakawagara et al., 2005

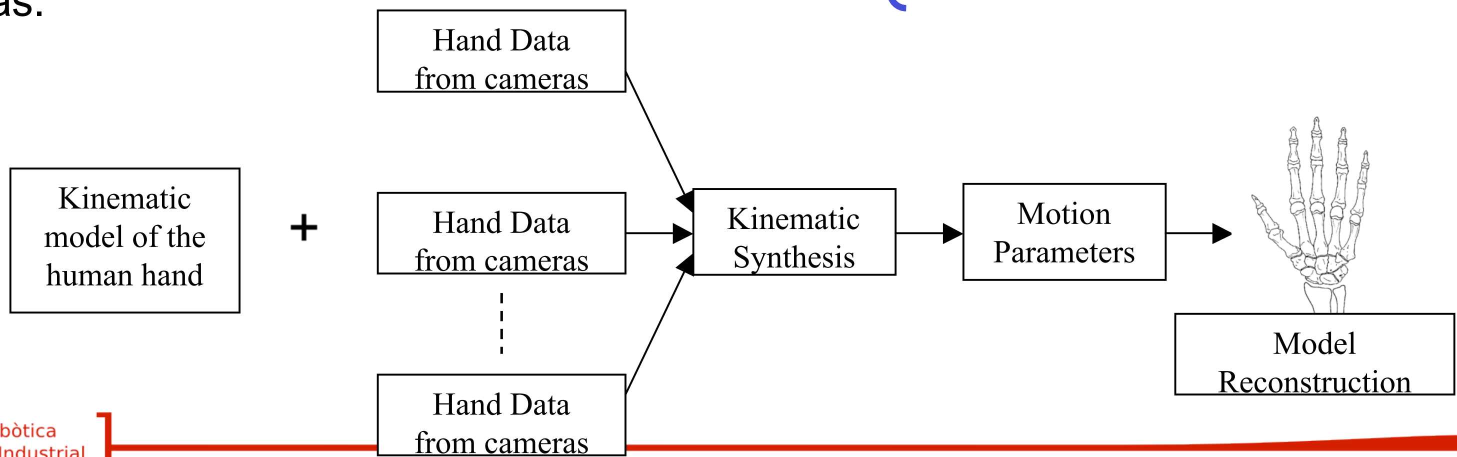
Motion capture methods:

Geometrical hand-shape models (motion library, thinning, solid fitting)
Pose estimation using kinematic hand models (full rotation at each joint)

Kinematic Synthesis Approach

Use kinematic synthesis to define the skeleton structure and the joint angles for a given motion.
3-D motion obtained from a set of video cameras.

→
- Non-intrusive system
- Relatively cheap
- Applicable to analyze other moving systems
- Adaptable to human variability



Kinematic Model of the Hand

Anatomy of the Human Hand: Degrees of Freedom

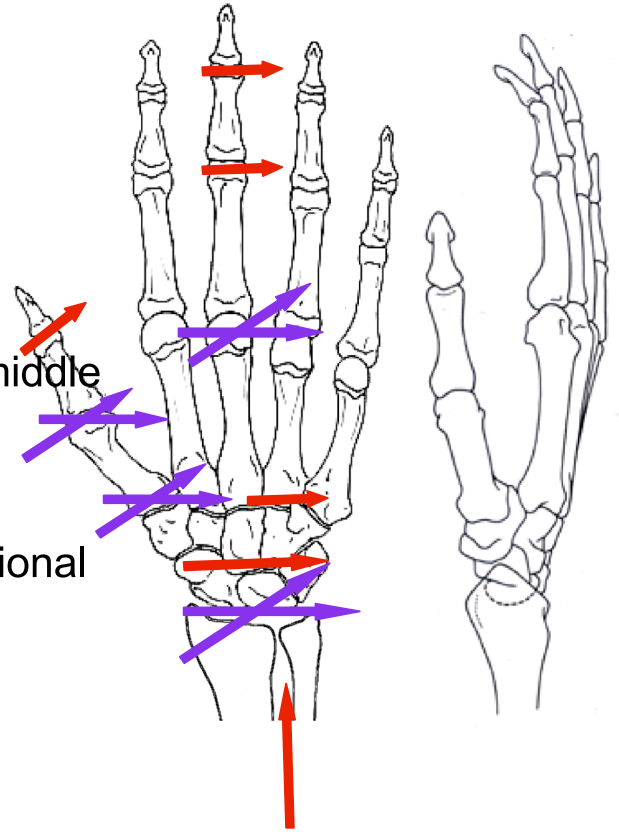
Joints are formed at the surface of relative motion between two bones
An accurate description of the relative motion uses the geometry and conjugation of the rubbing surfaces

Classification of hand joints:

- 1 DOF joints (rotation about an axes)
- 2 DOF joints (rotation about 2 axes)
- Immobile elements: Distal carpal bones, CMC, index and middle finger.

Each finger is considered as a serial chain

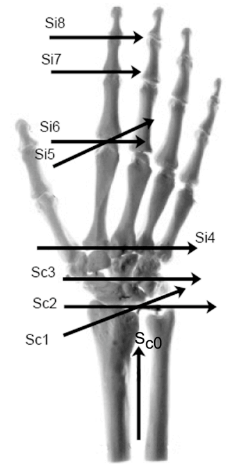
- Degrees of Freedom: each finger has 5 D.O.F, with 3 additional common D.O.F.



Kinematic Model of the Hand

Dual Quaternion Kinematic Model

Index, Middle, Ring and Little Finger



$$\hat{Q}_{ind} = \hat{S}_{c0}(\theta_0)\hat{T}_{c12}(\theta_1,\theta_2)\hat{S}_{c3}(\theta_3)\hat{T}_{i56}(\theta_5,\theta_6)\hat{S}_{i7}(\theta_7)\hat{S}_{i8}(\theta_8)$$

$$\hat{Q}_{mid} = \hat{S}_{c0}(\theta_0)\hat{T}_{c12}(\theta_1,\theta_2)\hat{S}_{c3}(\theta_3)\hat{T}_{m56}(\theta_5,\theta_6)\hat{S}_{m7}(\theta_7)\hat{S}_{m8}(\theta_8)$$

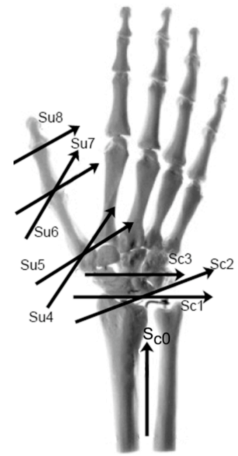
$$\hat{Q}_{thir} = \hat{S}_{c0}(\theta_0)\hat{T}_{c12}(\theta_1,\theta_2)\hat{S}_{c3}(\theta_3)\hat{S}_{t4}(\theta_4)\hat{T}_{t56}(\theta_5,\theta_6)\hat{S}_{t7}(\theta_7)\,\hat{S}_{t8}(\theta_8)$$

$$\hat{Q}_{four} = \hat{S}_{c0}(\theta_0)\hat{T}_{c12}(\theta_1,\theta_2)\hat{S}_{c3}(\theta_3)\hat{S}_{f4}(\theta_4)\hat{T}_{f56}(\theta_5,\theta_6)\hat{S}_{f7}(\theta_7)\,\hat{S}_{f8}(\theta_8)$$

Thumb Finger

$$\hat{Q}_{thum} = \hat{S}_{c0}(\theta_0)\hat{T}_{c12}(\theta_1,\theta_2)\hat{S}_{c3}(\theta_3)\hat{T}_{t45}(\theta_4,\theta_5)\hat{T}_{t67}(\theta_6,\theta_7)\hat{S}_{t8}(\theta_8).$$

Total model complexity

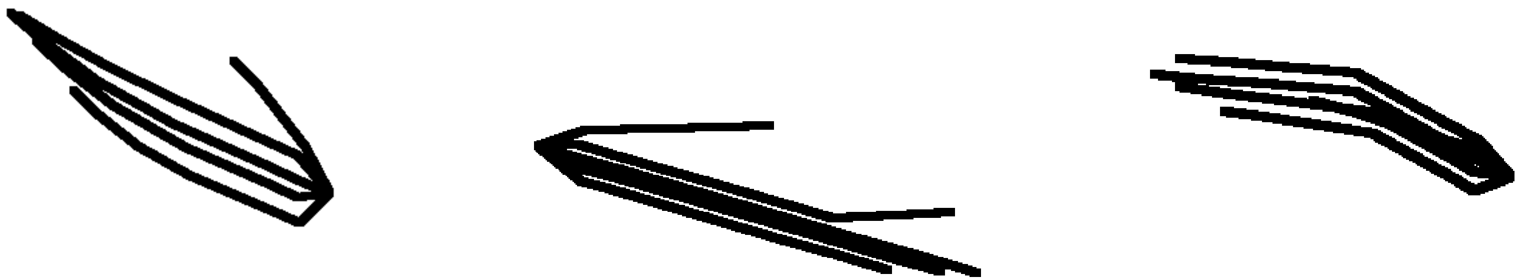


Our hand model has 14 revolute joints, with 4 structural parameters each and 7 universal joints, with 6 structural parameters each.
28 joint variables (rotations qi),
98 structural parameters to define the joint axes Si.

Kinematic Model of the Hand

Motion Simulation

Hand parameters taken from anatomy literature.
Define joint trajectories for the 27 joint variables.
Model was implemented in Maple to create kinematic simulation.
Accurate geometry (angles and distances between joints) need not be correct; to be refined with synthesis process.



Kinematic Synthesis of the Human Hand

For this application, we require precise values for the location of the joints and the joint rotations.

Kinematic synthesis can be applied to dimension the human hand and to track its motion.

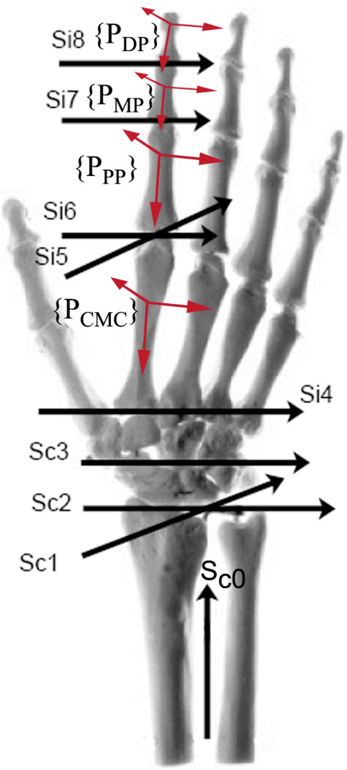
The displacement of each hand limb can be expressed as

- Lower arm P_{LA}
- Palm, carpal area P_{CA}
- Palm, carpo-metacarpal area P_{CMC}
- Proximal phalanx P_{PP},
- etc.

The simultaneous solution for all joints in each ray presents some problems:

- Complexity (33 different positions, 264 nonlinear equations)
- Multiple solutions

Instead, we solve for each joint in a hierarchical process.



Kinematic Synthesis of the Human Hand

Hierarchical Process

Solve for the proximal radio-ulnar joint with the motion of the lower arm

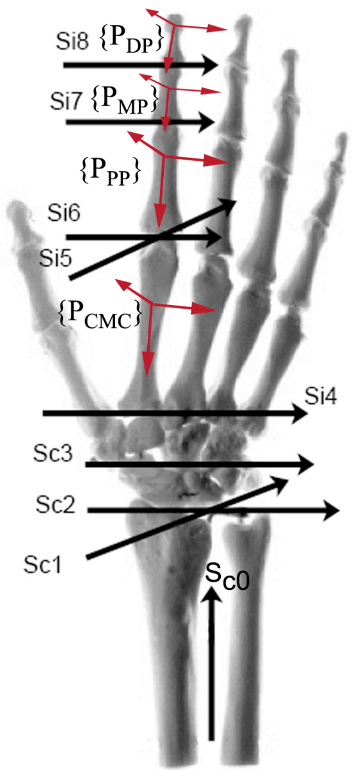
$$\hat{S}_{c0}(\theta_0^i) = \hat{P}_{LA}^i, \quad i = 2, \ldots n$$

→

Solve for the distal radio-ulnar joint and carpal joint using the motion of the palm carpal area

$$\hat{S}_{c0}(\theta_0^i)\hat{T}_{c12}(\theta_1^i, \theta_2^i)\hat{S}_{c3}(\theta_3^i) = \hat{P}_{CA}^i$$

$$\hat{T}_{c12}(\theta_1^i, \theta_2^i)\hat{S}_{c3}(\theta_3^i) = (\hat{P}_{LA}^i)^* \hat{P}_{CA}^i, \quad i = 2, \ldots n$$



Thumb

CMC joint (motion: thumb metacarpal area)
↓
MCP joint (motion: proximal phalanx)
↓
PP joint (motion: proximal phalanx)
↓
DP joint (motion: distal phalanx)

Index, middle

MCP joint (motion: proximal phalanx)
↓
PP joint (motion: proximal phalanx)
↓
MP joint (motion: middle phalanx)
↓
DP joint (motion: distal phalanx)

Third, fourth

CMC joint (motion: palm metacarpal area)
↓
MCP joint (motion: proximal phalanx)
↓
PP joint (motion: proximal phalanx)
↓
MP joint (motion: middle phalanx)
↓
DP joint (motion: distal phalanx)

Kinematic Synthesis of the Human Hand

Numerical Solver

- The numerical solution consists on the minimization of the distance between the real motion and the motion performed by the simplified joint,

$$\epsilon_i = \sqrt{\sum_{j=1}^{8} (\hat{Q}^i_j - \hat{P}^i_j)^2}$$

We use a Levenberg-Marquardt nonlinear least squares solver implemented in C++. (http://www.ics.forth.gr/~lourakis/levmar , Manolis Lourakis, Institute of Computer Science, Foundation for Research and Technology-Hellas, Heraklion, Crete).

For each joint, we have to solve a nonlinear system of equations. Tests show that the equations converge to a solution quickly.

```
cg = -0.1460830286e0 + s0x * sin (theta02 / 0.2e1);
cg0 = s0y * sin (theta02 / 0.2e1);
cg1 = s0z * sin (theta02 / 0.2e1);
cg2 = -0.9892723330e0 + cos (theta02 / 0.2e1);
cg3 = -0.2990407923e0 + s0x * sin (theta03 / 0.2e1);
cg4 = s0y * sin (theta03 / 0.2e1);
cg5 = s0z * sin (theta03 / 0.2e1);
cg6 = -0.9542403285e0 + cos (theta03 / 0.2e1);
cg7 = -0.4909037534e0 + s0x * sin (theta04 / 0.2e1);
cg8 = s0y * sin (theta04 / 0.2e1);
cg9 = s0z * sin (theta04 / 0.2e1);
cg10 = -0.8712138112e0 + cos (theta04 / 0.2e1);
cg11 = (c0y * s0z - c0z * s0y) * sin (theta02 / 0.2e1);
cg12 = (c0z * s0x - c0x * s0z) * sin (theta02 / 0.2e1);
cg13 = (c0x * s0y - c0y * s0x) * sin (theta02 / 0.2e1);
cg14 = (c0y * s0z - c0z * s0y) * sin (theta03 / 0.2e1);
cg15 = (c0z * s0x - c0x * s0z) * sin (theta03 / 0.2e1);
cg16 = (c0x * s0y - c0y * s0x) * sin (theta03 / 0.2e1);
cg17 = (c0y * s0z - c0z * s0y) * sin (theta04 / 0.2e1);
cg18 = (c0z * s0x - c0x * s0z) * sin (theta04 / 0.2e1);
cg19 = (c0x * s0y - c0y * s0x) * sin (theta04 / 0.2e1);
```

Results, middle finger joints **(3.45 GHz processor)**:

Axes	Error	Iteration	Time (sec)
S_{c0}	1.72259e-21	18	0.004
S_{c1} S_{c2}	2.00422e-19	247	0.012
S_{m4}	5.95462e-19	41	0.002
S_{m5} S_{m6}	2.59657e-18	634	0.024
S_{m7}	2.90753e-17	71	0.004
S_{m8}	7.91677e-17	51	0.002

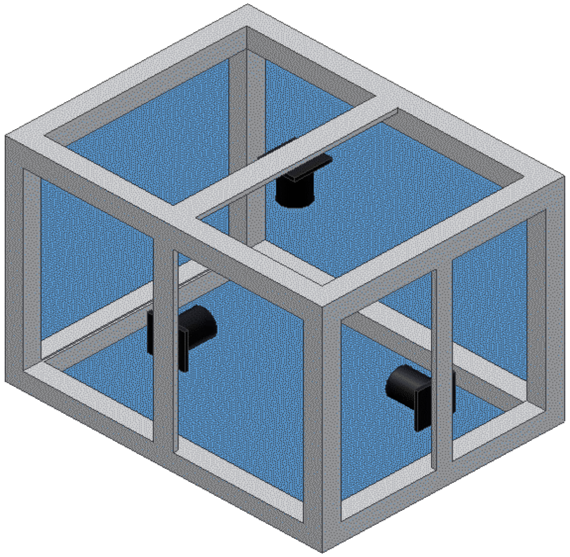
Kinematic Synthesis of the Human Hand

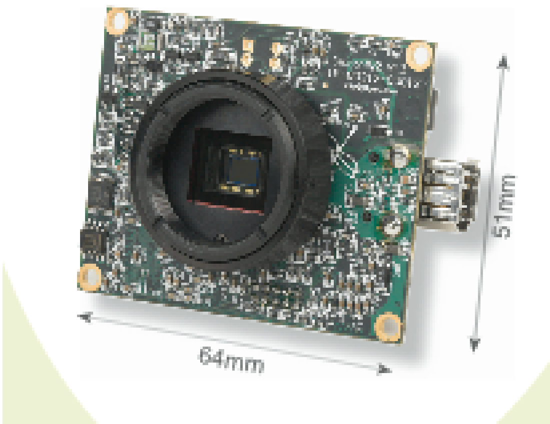
Image acquisition: experimental setup (under construction)

Actual motion capture takes place in a box of approx. dimensions 1mx1mx1m approx., equipped with three cameras.

Cameras: Dragonfly2, 648x488 resolution, 60 fps.

NI LabVIEW software for image acquisition and analysis.





Kinematic Synthesis of the Human Hand

Challenges of the image acquisition and processing

- 3-D model calculation from 2-D camera images.
- Motion segmentation (identify rigid bodies): Background subtraction, occultations, deformations, etc.
- Object classification (identify which rigid body corresponds to which limb)
- Tracking (identify the same rigid body in consecutive frames)
- We are dealing with more than 20 “rigid bodies” in the hand, some of them with very subtle motion and with visible deformation.
- For obtaining good results using kinematic synthesis, we need to be able to isolate the rigid motion associated to the skeleton.

Synthesis strategy

- Newton-based solver very fast when close to a solution.
- Usually, inverse kinematic solutions are not very close to each other.
- Use hierarchical synthesis to obtain an approximate solution
- Use the solution from the hierarchical synthesis as initial conditions for solving each finger as a complete RTRR or TTR serial chain.
- Input data: an easy feature of each finger (fingertip).