MATH 4441/5541: Introduction to Numerical Analysis I

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Homework Assignment #2

Due Tuesday September 17, 2013

1. Read Section 2.2 and show that $g(x) = 2^{-x}$ has a unique fixed point on $[1/3, 1]$. Use fixed point iteration to find an approximation to the fixed point accurate to within 10^{-4}, and using Corollary 2.5 to estimate the number of iterations required to achieve 10^{-4} accuracy, and compare this theoretical estimate to the number actually needed.

2. Read Section 2.4. Let $f \in C^m[a, b]$ has a zero of multiplicity $m \geq 2$ at $x^* \in (a, b)$.

   (a) Derive the Newton's iteration method to find the root x^* of the equation $f(x) = 0$.

   (b) Show that the Newton's method converges only linearly.

   (c) Let us modify the Newton's iteration method as follows:

   $$x_{k+1} = x_k - m\frac{f(x_k)}{f'(x_k)}.$$

   Show the above scheme converges quadratically.

3. Write a *MATLAB* Program for the Newton's method for solving the following nonlinear system of equations:

   $$\begin{cases} f_1(x, y) = 3x^2 - y^2 & = & 0 \\ f_2(x, y) = 3xy^2 - x^3 - 1 & = & 0. \end{cases}$$

   with the initial guess $(1, 1)$. Use `format long` and stop when both components of two successive iterates agree to 12 decimal places. Display the approximations given and the function values at each iteration. (You may check your answer using *Matlab* subroutine `fsolve`, see Matlab help page as needed.)