MATH 4441/5541: Introduction to Numerical Analysis I

Instructor: Yunrong Zhu

Homework Assignment #4

Due Thursday Oct. 24, 2013

1. Read Section 4.1, and do the following:

   (a) Derive an $O(h^4)$ five point formula to approximate $f'(x_0)$ that uses $f(x_0-h)$, $f(x_0)$, $f(x_0+h)$, $f(x_0+2h)$, and $f(x_0+3h)$. [*Hint*: Consider the expression $Af(x_0-h)+Bf(x_0+h)+Cf(x_0+2h)+Df(x_0+3h)$. Expand in fourth Taylor polynomials, and choose A, B, C, and D appropriately.]

   (b) Find an error expression (at x_0) for the numerical differentiation formula

   $$f'(x_0) \approx \frac{1}{2h}[4f(x_0+h) - 3f(x_0) - f(x_0+2h)].$$

2. Verify that Simpson's rule is exact for all polynomials of degree ≤ 3 by verifying it holds exactly for $f(x) = 1$, x, x^2, and x^3.

3. Construct Gaussisan-type quadrature formulas:

   (a) $\displaystyle\int_0^1 \frac{1}{\sqrt{x}} f(x)dx \approx A_0 f(x_0) + A_1 f(x_1)$.

   (b) $\displaystyle\int_{-1}^1 f(x)dx \approx af(-1) + bf(1) + cf'(-1) + df'(1)$.

4. Mimic the following subroutine for the composite middle-point rule, and write MATLAB subroutines for the composite Trapezoidal rule and composite Simpson's rule. Use these three subroutines to compute the integral $\int_0^1 \sqrt{x}\ln x \, dx = -\frac{4}{9}$.

```
function [ val ] = intcm( FUN, a, b, n )
%INTCM integral of the given function using the composite
%      middle point rule
%
%   [val] = intcm(FUN, a, b, n) find the integral of the given function
%   FUN on the interval [a, b] using n subintervals.
%   FUN must be a function handle.
%
%   Example
%     clear all
```

```
11 %      a = 0;
12 %      b = 1;
13 %      f = @(x) sqrt(x);
14 %      n = 100;
15 %      [val] = intcm(f, a, b, n);
16 %
17 %   Copyright (C) Yunrong Zhu, 2013.
18
19 h = (b−a)/n;     % step size
20
21 midx = a+h/2 : h: b;            % middle points
22 val = 0;
23 for i=1:n
24     val = val + feval(FUN, midx(i));
25 end
26 val = h*val;
27
28 %% Line 22−26 can be replaced by the following one line command,
29 %  which is more efficient.
30
31 %val = h*sum(feval(FUN, midx));
32 end
```

(a) Test these two quadrature rules using different choice of h, compare with the exact solution $-\frac{4}{9}$. Plot the dependence of the error with respect the step size h.

(b) Is there any minimal step size h such that the error can not be reduced?

(c) Compare the results with the MATLAB subroutine: `integral`. Type `help integral` for the usage of this subroutine.