

Backpropagation Homework

For this homework you will work through training a multi-layer perceptron using the back propagation algorithm on **one** training instance (the second training instance below, 0 1 \rightarrow 0). The first training instance is done for you to give an example. You should **use the updated weights** after training on the first instance when training on the second instance. Assume a bias of 1.

Example with first training pattern: BP-1) A 2-2-1 backpropagation model has initial weights as shown. Work through one cycle of learning for the following pattern(s). Assume 0 momentum and a learning constant of 1. Round calculations to 3 significant digits to the right of the decimal. Give values for all nodes and links for activation, output, error signal, weight delta, and final weights. Nodes 4, 5, 6, and 7 are just input nodes and do not have a sigmoidal output.

For each node calculate the following (show necessary equation for each).
Hint: Calculate bottom-top-bottom.

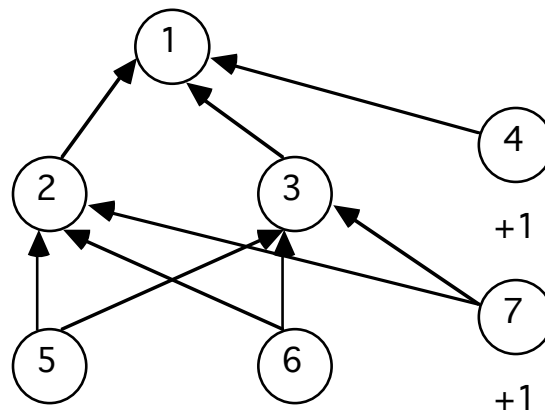
a =

o =

δ =

Δw =

w =



a) All weights initially 1.0

Training Patterns

1) 0 0 \rightarrow 1

2) 0 1 \rightarrow 0

Network Equations

BP-1)

$$\text{net}_2 = \sum w_i x_i = (1*0 + 1*0 + 1*1) = 1$$

$$\text{net}_3 = 1$$

$$o_2 = 1/(1+e^{-\text{net}_2}) = 1/(1+e^{-1}) = 1/(1+0.368) = .731$$

$$o_3 = .731$$

$$o_4 = 1$$

$$\text{net}_1 = (1*.731 + 1*.731 + 1) = 2.462$$

$$o_1 = 1/(1+e^{-2.462}) = .921$$

$$\delta_1 = (t_1 - o_1) o_1 (1 - o_1) = (1 - .921) .921 (1 - .921) = .00575$$

$$\Delta w_{21} = \eta \delta_j o_i = \eta \delta_1 o_2 = 1 * .00575 * .731 = .00420$$

$$\Delta w_{31} = 1 * .00575 * .731 = .00420$$

$$\Delta w_{41} = 1 * .00575 * 1 = .00575$$

$$\delta_2 = o_j (1 - o_j) \sum \delta_k w_{jk} = o_2 (1 - o_2) (\delta_1 w_{21} + \delta_3 w_{32}) = .731 (1 - .731) (.00575 * 1) = .00113$$

$$\delta_3 = .00113$$

$$\Delta w_{52} = \eta \delta_j o_i = \eta \delta_2 o_5 = 1 * .00113 * 0 = 0$$

$$\Delta w_{62} = 0$$

$$\Delta w_{72} = 1 * .00113 * 1 = .00113$$

$$\Delta w_{53} = 0$$

$$\Delta w_{63} = 0$$

$$\Delta w_{73} = 1 * .00113 * 1 = .00113$$

$$\text{Output: } O_j = f(\text{net}_j) = \frac{1}{1+e^{-\text{net}_j}}$$

$$f'(\text{net}_j) = \frac{\partial O_j}{\partial \text{net}_j} = O_j(1 - O_j)$$

$$\Delta w_{ij} (\text{general node}): C O_i \delta_j$$

$$\Delta w_{ij} (\text{output node}):$$

$$\delta_j = (t_j - O_j) f'(\text{net}_j)$$

$$\Delta w_{ij} = C O_i \delta_j = C O_i (t_j - O_j) f'(\text{net}_j)$$

$$\Delta w_{ij} (\text{hidden node})$$

$$\delta_j = \sum_k (\delta_k \cdot w_{jk}) f'(\text{net}_j)$$

$$\Delta w_{ij} = C O_i \delta_j = C O_i \left(\sum_k (\delta_k \cdot w_{jk}) \right) f'(\text{net}_j)$$

