Homework 10
CS 3385

1. From page 364, show that equation (15.4) follows from equation (15.3) and the initial condition $T(0) = 1$. This is most easily done using induction.

2. Show, by means of a counterexample, that the following "greedy" strategy does not always determine an optimal way to cut rods. The value per inch of a rod of length i (in inches) is p_i/i. The greedy strategy for a rod of length n makes its first cut of length i, where $1 \leq i \leq n$ and where i has the highest maximum price per inch. It then continues by applying the greedy strategy to the remaining piece of length $n - i$. Use the p_i values in the table given on page 360 of the textbook and $n = 4$ for your counterexample. Show which cuts are made and how much revenue they result in.

3. Consider a modification of the rod-cutting problem in which, in addition to a price p_i for each rod, each cut incurs a fixed cost of c. The revenue associated with a solution is now the sum of the prices of the pieces minus the costs of making the cuts. How would you modify the dynamic-programming algorithm (memoizedCutRodAux on pg. 366) to solve this modified problem?

4. Draw the recursion tree for the `mergeSort` procedure on an array of 16 elements. Explain why memoization fails to speed up a good divide-and-conquer algorithm such as `mergeSort`.

5. Suppose that in the rod-cutting problem we also have limit l_i on the number of pieces of length i that we are allowed to produce, for $i = 1, 2, \ldots, n$. Show that the optimal-substructure property no longer holds. You can show this by example.

6. Consider the following two DNA sequences:

   ```
   ATCC
   ACGC
   ```

   An optimal alignment will look something like this (this is an example of the format; it is not an optimal alignment):

   ```
   A TC C
   AC GC
   ```

   Give the optimal alignment and total cost for the two sequences given the following costs:

   (a) gap insertion = 2; mismatch = 1; match = 0
   (b) gap insertion = 1; mismatch = 2; match = 0