Homework 11
CS 3385

1. Given activities in random order, what is the running time (in asymptotic notation) of optimal activity selection using the `greedyActivitySelector` algorithm given on page 421?

2. Suppose that instead of always selecting the first activity to finish, we instead select the last activity to start that is compatible with all previously selected activities. Describe how this approach is a greedy algorithm, and prove that it yields an optimal solution. Your proof should follow the approach given in the Activity Selection video.

3. Not just any greedy approach to the activity-selection problem produces a maximum-size set of mutually compatible activities. Give examples to show that each of the following approaches does not work. Give your answers using something like the following format:

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   (a) Select the activity of least duration from among those that are compatible with previously selected activities.
   (b) Select the compatible remaining activity with the earliest start time.

4. Suppose that we have a set of activities to schedule among a large number of lecture halls, where any activity can take place in any lecture hall. We wish to schedule all the activities using as few lecture halls as possible. Give an efficient greedy algorithm to determine which activity should use which lecture hall. You do not need to prove correctness.

5. Prove that the fractional knapsack problem has the greedy-choice property.

6. Zeke plans to backpack across the Sawtooth Mountain Range in Idaho. He can carry two liters of water, and he can hike m miles before running out of water. Zeke will start with two full liters of water. His map shows all the places along the trail at which he can refill his water from streams (don't forget to filter, Zeke!) and the distances between these locations. The goal is to minimize the number of water stops along his route across the mountain range. Give an efficient greedy method by which he can determine which water stops he should make. Prove that your strategy yields an optimal solution, and give its running time. To prove that the problem has optimal substructure, follow the approach on page 416 in the paragraph beginning with "the usual cut-and-paste argument...".

7. Consider the set of frequencies that correspond to the first 8 Fibonacci numbers:
   `a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21`

   (a) What is an optimal Huffman code for these frequencies?
   (b) Generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers.