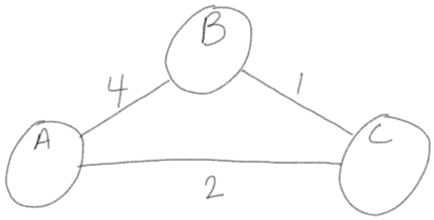
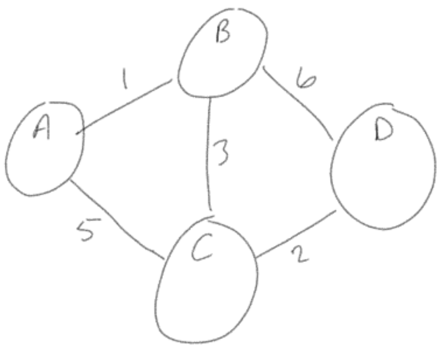
Homework 16
CS 3385

1. Run Dijkstra's algorithm on the following graph using vertex A as the source vertex. Use a table like what is shown in the video and show the graph labeled with d and pi values.



2. Run Dijkstra's algorithm on the following graph using vertex A as the source vertex. Use a table like what is shown in the video and show the graph labeled with d and pi values.



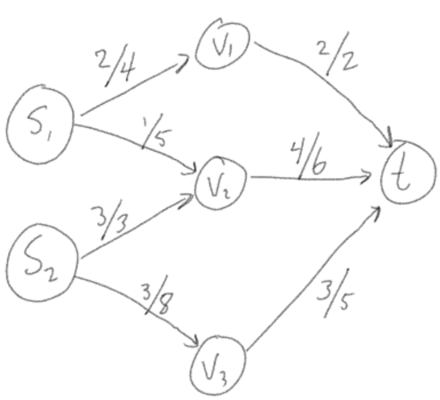
3. Give a simple example of a graph with negative-weight edges for which Dijkstra's algorithm produces incorrect answers. Run the algorithm on the graph and indicate the failure.

4. Suppose we change line 4 of Dijkstra's algorithm to the following:

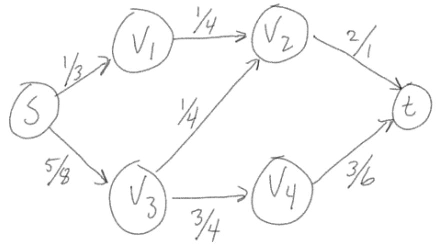
   ```
   4   while |Q| > 1
   ```

   This change causes the while loop to execute $|V| - 1$ times instead of $|V|$ times. Is this proposed algorithm correct? Why or why not?

5. Given the following network, show an equivalent flow network with a single source. Give the value of the flow.



6. Note any errors in the following flow.

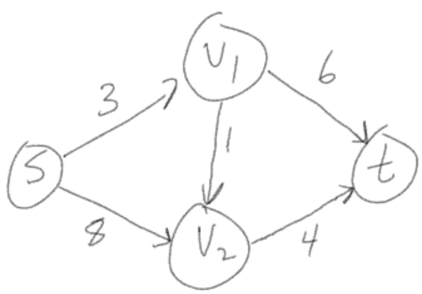


7. Computing cuts on directed flow graphs requires the following special considerations:

   - For capacity, we count only the capacities of edges going from S to T, ignoring edges in the reverse direction.
   - For flow, we consider the flow going from S to T minus the flow going in the reverse direction from T to S.

   With this in mind and looking at Figure 26.1(b) in the textbook, what is the flow across the cut $(\{s, v_2, v_4\}, \{v_1, v_3, t\})$? What are the flow and capacity of this cut?

8. In the following graph only the capacity for each edge is labeled. Show the min-cut of the capacity of the graph. Based on theorem 26.6 (the max-flow min-cut theorem) give the max flow value.



9. This problem treats the city of Reduct, a city created from scratch by forward-thinking people.

   (a) City designers are working on the layout of Reduct. Instead of laying out the streets in a grid system, they have decided to designate certain landmarks (e.g. city hall, grocery store, gas station, library) and make roads that (directly or indirectly) connect all of the landmarks. They want to pave as possible (to save money), so they have asked you to design a network of roads that connect all landmarks together with as little road as possible. Show that you can solve this problem in polynomial time by reducing it to a known polynomial time algorithm.

   (b) Two generations later, Reduct has grown into a bustling city. The minimal road network you designed has grown into a sophisticated network with many roads providing excellent transportation for city-dwellers. City leaders have decided to hold a 50th anniversary parade. They would like the parade to follow a route that visits each of the original landmarks and ends up where it started. Since the parade entries will cover the entire road, each road segment can only be on the parade route once. Parade designers have asked your granddaughter (who is also an excellent engineer like you) to design just such a route. Show that this will be a difficult task to solve exactly (i.e., there is no known polynomial time solution) by reducing a known NP-Complete problem to the parade route-finding problem. *Hint*: We've only talked about one or two NP-Complete problems; you'll reduce one of those to this problem.