1. [40 points] For the following signal:

\[
x(t) = \frac{4}{4t^2 + 1}
\]

find the essential bandwidth \( B \) (in Hz) of \( x(t) \), such that energy contained in the spectral components of \( x(t) \) at frequencies below \( B \) accounts for 95% of \( x(t) \)'s energy \( E_x \). Draw \( |X(f)|^2 \) and show \(-B\) and \( B\).
2. [40 points] Given the following system configuration, find the impulse and frequency responses, i.e. $h(t)$ and $H(\omega)$, such at $Y(\omega) = H(\omega)X(\omega)$, where $Y(\omega) = \mathcal{F}\{y(t)\}$ and $X(\omega) = \mathcal{F}\{x(t)\}$. Sketch $|H(\omega)|$ by hand, then confirm your sketch by comparing it with a separate plot generated using MATLAB.

\[ y(t) = \alpha(t) + \alpha(t - t_0) \]

Impulse Response $h(t) = \delta(t) + \delta(t - t_0)$

Frequency Response $H(\omega) = 1 + e^{-j\omega t_0}$
\[ = e^{j\omega t_0} \left[ e^{-j\omega t_0} + e^{-j\omega t_0} \right] \]
\[ = 2 \cos \left( \frac{\omega t_0}{2} \right) e^{-j\omega t_0} \]

$|H(\omega)| = 2 \left| \cos \left( \frac{\omega t_0}{2} \right) \right|$

3. [20 points] Show that Fourier transform of the auto correlation, $R_{xx}(\tau)$, of a power signal $x(t)$, equals the power spectral density (PSD) of $x(t)$.

(See Section 3.8.2, pp. 150-151 of Lathi & Ding, or the lecture recording on PSD vs. autocorrelation. The basis for this property is Parseval's theorem that defines $E_x$ where $x(t)$ is truncated signal of $x(t)$. Letting $T \to \infty$ shows that $R_{xx}(\tau) = \text{PSD of } x(t)$.)