Wi-Fi Assisted Two-Hop Relay Probing in WiGig Device to Device Networks

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Abstract— Relaying is a key technology for millimeter wave communications to extend the range and to route around blockages. In practical implementation of relaying systems, probing is required to identify proper neighbor terminals which will serve as relays. There is however an inherent trade-off between relay probing and required overhead. In this paper, we consider WiGig (IEEE 802.11ad) devices, which are multiband capable with Wi-Fi support, and propose a Wi-Fi assisted relay probing for WiGig device-to-device networks. In the proposed scheme, Wi-Fi received signal strengths are used to pre-select the WiGig relays expected to maximize the spectral efficiency of the overall system. Then, only these pre-selected relays are used in online relay probing step. Simulation analysis demonstrate improvements in throughput and energy consumptions over existing relay probing schemes.

Index Terms— WiGig, Wi-Fi, Relay, D2D networks.

I. INTRODUCTION

Millimeter wave (mmWave) communications is considered as a promising enabler of fifth generation (5G) and beyond (B5G) networks [1-3]. The current IEEE 802.11ad standard and the upcoming IEEE 802.11ay standard defined a set of 60 GHz wireless network protocols also known as WiGig [2,3]. Due to its short-range directional transmissions, mmWave is best fitted to proximity device-to-device (D2D) communications [1]. To extend the range of mmWave communications and to route around blockages, the concept of relaying has been proposed [4-7]. In [4], the potential benefits of deploying outdoor mmWave relays are demonstrated in terms of the coverage probability. In [5], the coverage improvement of mmWave cellular networks through relaying is quantified. In [6], the impact of beamwidth and self-interference in mmWave relaying is studied.

Another design issue in relaying system is probing [7] which involves the process of identifying proper neighbor terminals which can serve as reliable relays. Since practical implementation of mmWave communications mandates the use of beamforming training (BT) with a non-negligible overhead, an inherent trade-off exists between relay probing and required overhead. Based on the optimal stopping theory, the work in [7] proposed a throughput-optimal relay probing scheme. In this scheme, the relay probing process is conducted online one by one based on real-time channel measurements and terminated once the expected optimized spectral efficiency threshold is obtained. Towards that, a fixed-point equation is used to characterize the expected optimal spectral efficiency threshold for a given link availability and probing overhead. However, the scheme given in [7] has several drawbacks which prevent its practical realizations. In [7], the exact distribution of the spectral efficiencies including link availabilities of the relay links should be known beforehand, which is impractical in real scenarios. Moreover, the relay probing process is stopped once the pre-defined number of probed relays is reached, while there might be other non-probed relays having better spectral efficiencies than the probed ones. It is stated that as we increase the number of probed relays, the probability of obtaining better spectral efficiency is increased [7].

In an effort to further improve the achievable throughput while overcoming the drawbacks of the scheme given in [7], we propose a relaying scheme with an offline relay pre-selection mechanism before conducting the online probing process. The offline relay pre-selection allows only those pre-selected relays to participate in the real-time probing process. This will decrease the number of probed relays, effectively resulting in higher throughput and lower energy consumptions. For the pre-selection phase, we take advantage of the Wi-Fi interfaces noting that IEEE 802.11ad standardized WiGig devices are multiband capable [1-3]. In the proposed system, Wi-Fi received signal strengths (RSSs) from source (S)-to-relay (R) and from relay (R)-to-destination (D) collected at the source are used to pre-select the “good” WiGig relays. More precisely, Wi-Fi RSSs from S-R and R-D are used to estimate the separation distances between S-R and R-D in addition to their probability density functions (PDFs). Based on these values, the probability of WiGig signal-to-noise power ratio (SNR) of both S-R and R-D links are evaluated for every candidate S-R-D link. Then, via a proposed probabilistic indicator, the group of relays expected to maximize the spectral efficiency of the S-R-D link are nominated for the online WiGig relay probing process, and
the best relay among them is selected for constructing the WiGig D2D relay link. Simulation results demonstrate the throughput gains in addition to the reduction in energy consumptions of the proposed scheme over those in [7].

The rest of the paper is organized as follows: In Section II, we present the system model and problem formulation. In Section III, we introduce the proposed WiGig relay probing scheme. In Section IV, we present simulation results to demonstrate the performance of the proposed scheme followed by the conclusion in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we first provide an overview of Wi-Fi and WiGig link models, then present the problem formulation of WiGig D2D relaying.

A) Link Models

The Wi-Fi received power at a separation distance \( d \) between two Wi-Fi devices operating at 5.25 GHz is expressed as [8]:

\[
P_{R}^w[dBm] = P_{T}^w[dBm] - L_w(d_0) - 10 \alpha_w \log_{10}(d) - \chi_w, \quad (1)
\]

where \( P_{T}^w \) and \( P_{R}^w \) are the transmitter (TX) and receiver (RX) Wi-Fi powers in dBm. \( L_w(d_0) \) is the path loss at a reference distance \( d_0 = 1 \) m. \( L_w(d_0) \) is equal to 47.2 [dB] based on the real measurements campaign introduced in [8]. \( \alpha_w = 2.32 \) is the path loss exponent and \( \chi_w = N(0, \sigma_w) \) is the dB shadowing term with zero mean and standard deviation of \( \sigma_w = 6 \) dB [8].

The WiGig received power considering beamforming gain and blockage effect can be expressed as [1,7]

\[
P_{R}^g = \eta(P_{LOS}(d)) P_{T}^g A_{TX}(\theta) A_{RX}(\phi) / L_g(d), \quad (2)
\]

where \( \eta(P_{LOS}(d)) \) is a Bernoulli random variable representing the blockage effect with parameter \( P_{LOS}(d) \), which indicates the distance-dependent line-of-sight (LOS) probability. \( P_{T}^g \) is the WiGig TX power in Watt. \( A_{TX}(\theta) \) and \( A_{RX}(\phi) \) are the TX and RX beamforming gains respectively, where \( \theta \) and \( \phi \) are the angle of departure (AoD) and angle of arrival (AoA). Assuming 2D steerable antenna model with Gaussian main loop profile [9], beamforming gains are given by

\[
A_{TX}(\theta) = A_0 e^{-\frac{4\ln(2)}{\theta_{3\text{dB}}}^2}, \quad A_0 = \left( \frac{1.162\theta_{3\text{dB}}}{\sin^2(\theta_{3\text{dB}})/2} \right)^2, \quad (3)
\]

where \( \theta_{3\text{dB}} \) and \( A_0 \) are the boresight angle of the TX beam, -3dB beamwidth and the maximum antenna gain respectively. For \( A_{RX}(\phi) \), we can use (3) except that \( \theta \) is replaced by \( \phi \) and \( \theta_{3\text{dB}} \) is replaced by \( \phi_{3\text{dB}} \), which is the boresight angle of the RX beam.

In (2), \( L_g(d) \) is the path loss and assuming that only LOS is considered [1,7], as it is the most dominant in mmWave communications, it is given by

\[
10 \log_{10}(L_g(d)) = \beta_g + 10 \alpha_g \log_{10}(d) + \chi_g, \quad (4)
\]

where \( \beta_g \) is the path loss at a reference distance, \( \alpha_g \) is the path loss exponent and \( \chi_g = N(0, \sigma_g) \) is log-normal shadowing term with zero mean and standard deviation of \( \sigma_g \).

Blockages are modeled as cylinders and spatially distributed according 2D homogenous Poison point process (PPP) [5]. Accordingly, the blockage parameter \( P_{LOS}(d) \) is given as [5]

\[
P_{LOS}(d) = \nu e^{-\omega d}, \quad (5)
\]

where \( \nu = e^{-\pi \Delta \lambda \Omega^{2}} \) and \( \omega = 2 \Delta \Omega \lambda \), \( \Delta \) is the obstacles thinning factor, \( \lambda \) denotes the obstacles density and \( \Omega \) denotes the obstacle radius, and \( E[\cdot] \) denotes the expectation operation [5].

B) WiGig D2D Relaying

WiGig devices are multiband capable, i.e., they contain both Wi-Fi and WiGig interfaces. In the proposed system, if the direct link between \( S \) and \( D \) is blocked or cannot be constructed, WiGig D2D two-hop relaying is utilized. The relaying scheme is designed to find out the best relay among \( N_p \) probed relays, which maximizes the overall throughput of the \( S-R-D \) link. Mathematically speaking, this can be expressed as:

\[
R^* = \max_{1 \leq i \leq N_p} (\psi_{S-R_i-D}) = \max_{1 \leq i \leq N_p} \left( \frac{W_T D_T \gamma_{S-R_i-D}}{T_p^{N_p} + T_{DT}} \right), \quad (6)
\]

where \( \psi_{S-R_i-D} \) is the spectral efficiency in bit/sec/Hz of the \( S-R_i-D \) link, \( W \) is the signal bandwidth, \( T_{DT} \) is the time duration of the data transmission and \( T_p^{N_p} \) represents the time required for probing \( N_p \) relays.

Without loss of generality, we assume a half-duplex decode-and-forward relay strategy where time resources are equally divided between \( S-R_i \) and \( R_i-D \) links. Thus, \( \psi_{S-R_i-D} \) can be expressed as [7]

\[
\psi_{S-R_i-D} = \min(0.5 \log_2(1 + \gamma_{S-R_i}), 0.5 \log_2(1 + \gamma_{R_i-D})), \quad (7)
\]

In (7), \( \gamma_{S-R_i} = P_{T}^g \mu_{S-R_i}/\sigma_0 \) represents the WiGig SNR of the \( S-R_i \) relay link, where \( P_{T}^g \mu_{S-R_i} \) is the received WiGig power from \( S \)-to-\( R_i \) and \( \sigma_0 \) is the noise power. Same equation can be applied to calculate \( \gamma_{R_i-D} \).

First, the \( S-R_i \) relay link is probed for a time duration of \( \tau_g \) sec, and if it is not blocked, the \( R_i-D \) relay link is probed in consequence, otherwise another relay will be tested. Thus, \( T_p^{N_p} \) can be expressed mathematically as:

\[
T_p^{N_p} = \sum_{i=1}^{N_p} \tau_g \left( 1 + \eta(P_{LOS}(d_{S-R_i}) \right)), \quad (8)
\]

where \( \eta(P_{LOS}(d_{S-R_i}) \) is the Bernoulli random variable representing the blockage of the \( S-R_i \) link.

It is obvious from (6) that a trade-off exists between probing more relays, i.e., increasing \( N_p \), and maximizing \( \psi_{S-R_i-D} \). In [7], an optimal stopping rule is proposed to find out the optimal...
$N_p^*$ expected to maximize the average $Ψ_{S-R-D}$. The relay probing process is stopped once $N_p^*$ is reached although better spectral efficiency can be obtained by probing more relays other than $N_p^*$. Actually, the optimality of this scheme is based on performing online relay probing only, which prevents testing all available relay space due to the aforementioned tradeoff. To further improve the performance, we propose a modified scheme where a small number of “good” relays, $N_g$ (among all candidate relays, $N_T$) expected to maximize the spectral efficiency of the $S-R-D$ link are selected in an offline phase. Only, these pre-selected relays will be used in the online relay probing process, i.e., $N_p = N_g$, and the best relay among them is selected for constructing the relay link. Thus, enhancements in throughput and energy consumptions can be obtained as demonstrated by the proposed WiGig relay probing scheme in the following section.

III. PROPOSED PROBING SCHEME

The proposed scheme mainly consists of two phases, namely; the offline WiGig relays pre-selection phase and the online relay probing and relay selection phase.

A) Offline WiGig Two-hop Relays Pre-Selection Phase

In this phase, the WiGig two-hop relays, expected to maximize the spectral efficiency of the $S-R-D$ link are enumerated. Towards that, the Wi-Fi RSSs from $S-R_i$, $P_{rS-R_i}$ and from $R_i-D$, $P_{rR_i-D}$, $1 \leq i \leq N_T$, are collected at the source node. It is noted that Wi-Fi transmission uses CSMA/CA MAC protocol, which avoids collisions with the other nearby Wi-Fi networks. Also, only short pilot frames are used in estimating the Wi-Fi RSS. Thus, collisions with the other surrounding Wi-Fi networks will be negligible and will not be considered for the sake of simplicity. Based on the collected Wi-Fi RSSs, the PDFs of both $γ_{S-R_i}$ and $γ_{R_i-D}$ are anticipated. Then, based on a proposed probabilistic metric, the number of relays, $R_s 1 \leq s \leq N_g$, expected to maximize the spectral efficiency of the $S-R-D$ link are pre-selected for online probing. In the followings, we consider the $S-R_i$ relay link for brevity of presentation. However, the same equations can be applied for the $R_i-D$ link. Based on (1), the expected value of the estimated distance, $\hat{d}_{S-R_i}$, between $S-R_i$ can be estimated as

$$ \mathbb{E}[\hat{d}_{S-R_i}] = 10 \frac{(C_1 P_{rS-R_i}^w)}{10 \sigma_w} = \frac{10 \alpha_w}{\ln 10} \frac{1}{d_{S-R_i}^2} \exp \left(-C_1 P_{rS-R_i}^w - 10 \sigma_w \frac{\log_{10}(d_{S-R_i})}{\alpha_w} \right), $$

(9)

where $C_1 = P_{rS-R_i}^w [\text{dB}] - L_{wp} (d_b)$, $d_{S-R_i}$ is a random variable representing the estimated distance between $S$ and $R_i$. The PDF of $\hat{d}_{S-R_i}$ can be then evaluated using random variable transformation (RVT) [10] as follows, see Appendix A.

$$ f_{\hat{d}_{S-R_i}} (\hat{d}_{S-R_i}) = \frac{1}{d_{S-R_i}^2 (\ln 10) / 2 \sigma_w^2} \exp \left(-C_1 P_{rS-R_i}^w - 10 \sigma_w \frac{\log_{10}(d_{S-R_i})}{\alpha_w} \right) \frac{(10 \alpha_w / \ln 10)}{d_{S-R_i}^2 2 \sigma_w^2}, $$

(10)

Also, $\mathbb{E}[\hat{d}_{S-R_i}]$ in (9) is used to estimate the Bernoulli parameter in (5) as follows:

$$ \hat{p}_{\text{LOS}} (\mathbb{E}[\hat{d}_{S-R_i}]) = e^{-\omega} \mathbb{E}[\hat{d}_{S-R_i}]. $$

(11)

Based on (10) and (11), the PDF of the WiGig received power of the $S-R_i$ relay link, i.e., $f_{P_{rS-R_i}} (P_{rS-R_i})$ can be evaluated as well as $f_{\gamma_{S-R_i}}(\gamma_{S-R_i})$, where $\gamma_{S-R_i} = P_{rS-R_i}^w / \delta_0$. From (2), and by using RVT [10], $f_{P_{rS-R_i}} (P_{rS-R_i})$ can be expressed as:

$$ f_{P_{rS-R_i}} (P_{rS-R_i}) = \frac{c_2}{(P_{rS-R_i}^w)^2} f_{\gamma_{S-R_i}} (\gamma_{S-R_i}), $$

(12)

where $c_2 = 2 \gamma_{S-R_i} / \pi \hat{p}_{\text{LOS}} (\mathbb{E}[\hat{d}_{S-R_i}]).$ Here, it is assumed that both $S$ and $R_i$ are using their maximum antenna gains for constructing the relay link. The detailed derivation of $f_{\gamma_{S-R_i}}(\gamma_{S-R_i})$ given in Appendix B. Using $f_{P_{rS-R_i}} (P_{rS-R_i})$ given in (12), $f_{\gamma_{S-R_i}}(\gamma_{S-R_i})$ can be easily calculated in accordance. Also, same methodology can be used to calculate $f_{P_{rR_i-D}} (P_{rR_i-D})$ and $f_{\gamma_{R_i-D}}(\gamma_{R_i-D})$.

Based on $f_{\gamma_{S-R_i}}(\gamma_{S-R_i})$, and $f_{\gamma_{R_i-D}}(\gamma_{R_i-D})$, the following performance metric is calculated for every candidate relay:

$$ \Gamma_{S-R_i-D} = \min \left\{ P_{Y_{S-R_i-D}}, \text{if } \gamma_{S-R_i} \geq \gamma_{th}, P_{Y_{R_i-D}}, \text{if } \gamma_{R_i-D} \geq \gamma_{th} \right\}, $$

(13)

$$ \Gamma_{S-R_i-D} = \min \left\{ P_{Y_{S-R_i-D}}, \text{if } \gamma_{S-R_i} \geq \gamma_{th}, P_{Y_{R_i-D}}, \text{if } \gamma_{R_i-D} \geq \gamma_{th} \right\}, $$

(14)

where $\gamma_{th}$ is the minimum spectral efficiency that can be provided by a WiGig link [2,3]. Thus, $\Gamma_{S-R_i-D}$ can be considered as a probabilistic metric of the coverage probability that probably provided by an $S-R_i-D$ relay link. After evaluating $\Gamma_{S-R_i-D}$ for all $N_T$ relays, the source node sorts them in descending order and selects the group of “good” relays with high values of $\Gamma_{S-R_{i,D}}$ as

$$ R_s = \text{Sort} (\Gamma_{s,R-D})_{1 \leq s \leq N_s} $$

(15)

A) Online Relay Probing and Relay Selection

In this phase, real time relay probing takes place through the pre-selected $N_g$ relays, and the best relay among them, i.e., $R_s$ is selected to construct the $S-R-D$ relay link. This can be expressed mathematically as:

$$ R_s = \max_{1 \leq s \leq N_s} (\Psi_{S-R-D}) = \max_{1 \leq s \leq N_s} \left( \frac{\max_{1 \leq s \leq N_T} W_{DT} \Psi_{S-R_{s-D}}}{\Omega_{\text{Off}} + \Omega_{\text{On}} + T_{DT}} \right). $$

(15)
where $\psi_{S-R_s-D}$ and $T_p^{N_S}$ are evaluated as given in (7) and (8), respectively. Compared to (6), $T_p^{NT}$ is added to the dominator of (15) to represent the time consumed by the offline phase related to Wi-Fi messages over all $N_r$ candidate relays, which can be expressed simply as $T_{\text{Off}}^{Ny} = 2N_r \tau_w$, $\tau_w$ indicates the time consumed by Wi-Fi messages such as RSS measurement request / response frames, and 2 is used to consider the Wi-Fi messages of both $S-R_i$ and $R_i-D$ relay links. $T_{\text{Off}}^{Ny}$ typically consumes negligible time compared to $T_p^{N_S}$ as explained in [1].

### IV. Simulation Analysis

In this section, we present simulation results to demonstrate the throughput performance of the proposed WiGig D2D two-hop relaying scheme. We consider a square room with each side of 25 m. Table I summarizes simulation parameters where 100 relays are uniformly distributed in the area between the source node located at the origin and the destination node located at the opposite corner.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$</td>
<td>2.16 GHz [1]</td>
</tr>
<tr>
<td>$\tau_g$</td>
<td>50 msec [1]</td>
</tr>
<tr>
<td>$P_t^w$</td>
<td>20 dBm [1]</td>
</tr>
<tr>
<td>$P_t^s$</td>
<td>54.9 dB [1]</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>14.6 dB [1]</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>1 [5]</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>uniform [0.3 - 0.6] m [5]</td>
</tr>
<tr>
<td>$T_{DTr}$</td>
<td>1 sec [1]</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>3.6 usec [1]</td>
</tr>
<tr>
<td>$P_t^r$</td>
<td>10 dBm [1]</td>
</tr>
<tr>
<td>$\Delta_{g}$</td>
<td>3.88 [1]</td>
</tr>
<tr>
<td>$\delta_g$</td>
<td>-73 dBm</td>
</tr>
<tr>
<td>$\psi_{\min}$</td>
<td>27.5 Mbps [1,2]</td>
</tr>
</tbody>
</table>

In Table I, we consider a square room with each side of 25 m. Table I summarizes simulation parameters where 100 relays are uniformly distributed in the area between the source node located at the origin and the destination node located at the opposite corner.

In Fig. 1, we show the tradeoff between the number of online probed relays and average throughput at different values of blocking density ($\lambda$). It is observed that as we increase the target spectral efficiency, more relays should be probed, and the average throughput is decreased in consequence. This motivates the design of proper relay probing schemes.

In Fig. 2, we present the average throughput versus different values of $\lambda$. It is observed that the proposed scheme has better average throughput in comparison to [7] at all $\lambda$ values under consideration. At $\lambda = 0$, although both schemes almost use the same number of probed relays, 41% enhancement in the average throughput is obtained using the proposed scheme thanks to the beforehand good relays pre-selection.

In Fig. 3, we present the average number of probed relays and compare with that in [7]. It is observed that our scheme needs a smaller value especially for high values of $\lambda$. This comes from the offline pre-selection phase, which considers the expected $P_{\text{LOS}} \mathbb{E}[\hat{d}_{S-R_i}]$ when selecting the good relays for the online relay probing phase. For example, if $\lambda = 0.1$ is considered, 40% reduction in the average number of propped relays is obtained using the proposed scheme.

Fig. 4 represents the average energy consumption comparisons of both schemes in m joule. For the scheme proposed in [7], the energy consumption is calculated as:

$$
\mathcal{E}_c = P_t^\theta \left( T_{p}^{N_S} + T_{DTr} \right),
$$

(16)

However, for the proposed scheme, the energy consumption can be calculated as:

$$
\mathcal{E}_p = P_t^w T_{\text{Off}}^{Ny} + P_t^\theta \left( T_{p}^{N_r} + T_{DTr} \right),
$$

(17)

where $P_t^\theta$ and $P_t^w$ in (16) and (17) are in Watt and $N_p^*$ is given in Fig. 3 for different values of $\lambda$. In (17), the term $P_t^w T_{\text{Off}}^{Ny}$ is added to represent the energy consumption coming from Wi-Fi signaling used for RSS measurements. As shown in Fig. 4, the trends of average energy consumptions of both schemes are similar to their trends of average number of probed relays. This means that $T_{p}^{N_S}$ and $T_{p}^{N_r}$ are the dominant factors in (16) and (17), and the term $P_t^w T_{\text{Off}}^{Ny}$ has a negligible effect on the consumed energy of the proposed scheme. From Fig. 4, at $\lambda = 0.1$, about 26.17% reduction in average energy consumption is obtained using the proposed scheme.

### V. Conclusion

In this paper, we proposed a novel WiGig D2D two-hop relaying. It consists of an offline phase to enumerate the number of good relays to be used for online relay probing and relay selection phase. Wi-Fi RSS is used as an enabler for the offline phase, by which Wi-Fi RSS is used to estimate the PDF of WiGig received SNR of each candidate relay link. This in turns is used to pre-select the good relays expected to maximize the spectral efficiency of the WiGig two-hop relaying. The best relay among the pre-selected ones is chosen for constructing the WiGig relay link after the process of online relay probing. The proposed scheme shows superior performance with respect to average number of probed relays, average throughput and average energy consumption over the existing scheme in the literature, which is based on online relay probing only.

### APPENDIX A: CALCULATION OF $f_{\hat{d}_{S-R_i}}(\hat{d}_{S-R_i})$:

From (1), the PDF of the estimated distance $\hat{d}_{S-R_i}$ based on the received Wi-Fi RSS and the log-normal shadowing term, can be written using RVT [10] as:

$$
f_{\hat{d}_{S-R_i}}(\hat{d}_{S-R_i}) =
\int_{x_w(\hat{d}_{S-R_i})} f_{x_w}(x_w) \left| \frac{\partial x_w}{\partial \hat{d}_{S-R_i}} \right| \mathbb{I}_{\{c_1 - P_t^{w,R_i} - 10\sigma_{\text{w}} \log_{10}(\hat{d}_{S-R_i}) \geq 0\}},
$$

(18)

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\[
    f_{t,g,s-R_i}(L_{g,s-R_i}) = \frac{(10\alpha_W/\ln 10)}{2\pi \sigma_g L_{g,s-R_i} \sigma_W} \exp\left(-\frac{(\ln t)^2}{2\sigma_g^2} + \frac{\left(C_1 - P_{t,g,s-R_i} - 10\alpha_W \log_{10}\left(\frac{L_{g,s-R_i}}{C_3}\right)\right)^2}{2\sigma_W^2}\right) dt
\]

where, \(C_1 = P_t^w [\text{dB}] - L_w(d_0)\), and

\[
    f_\chi_w(\chi_w) = \frac{1}{\sqrt{2\pi \sigma_w^2}} \exp\left(-\frac{\chi_w^2}{2\sigma_w^2}\right). \tag{19}
\]

Thus,

\[
    \left| \frac{\partial \chi_w}{\partial \tilde{d}_{S-R_i}} \right| = \frac{10\sigma_w}{\tilde{d}_{S-R_i} \ln 10}. \tag{20}
\]

Substituting (18) and (19) into (20), \(f_\tilde{a}_{S-R_i}(\tilde{d}_{S-R_i})\) can be written as given in (10).

**APPENDIX B: CALCULATION OF \(f_{L_{g,s-R_i}}(L_{g,s-R_i})\):**

From (4), \(L_{g,s-R_i}\) can be expressed in linear form as:

\[
    L_{g,s-R_i} = 10^{\beta_{g}} + 10^{\kappa} \chi_{g,LN} \left(\tilde{d}_{S-R_i}\right)^{a_{g}}, \tag{21}
\]

where \(\kappa = \frac{\ln(10)}{10}\) and \(\chi_{g,LN} \sim LN(0, \sigma_g)\) denotes a log-normal random variable with PDF \(f_{\chi_{g,LN}}(x) = \frac{1}{\sqrt{2\pi \sigma_g^2}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma_g^2}\right), x > 0\). From (21), \(L_{g,s-R_i}\) is a multiplication of two independent random variables, i.e., \(\chi_{g,LN}\) and \(\tilde{d}_{S-R_i}\). Let \(\mathcal{T} = (\tilde{d}_{S-R_i})^{a_{g}}\), thus, \(f_\mathcal{T}(\mathcal{T})\) can be written using RVT \([10]\) as:

\[
    f_\mathcal{T}(\mathcal{T}) = f_\tilde{a}_{S-R_i}(\tilde{d}_{S-R_i}) \left| \frac{\partial \tilde{a}_{S-R_i}}{\partial \mathcal{T}} \right|_{\tilde{d}_{S-R_i}, \mathcal{T} = \tilde{d}_{S-R_i}^{a_{g}}} = \frac{1}{a_{g}} \tag{22}
\]

Using \(f_\tilde{a}_{S-R_i}(\tilde{d}_{S-R_i})\) in (10), \(f_\mathcal{T}(\mathcal{T})\) can be written as:
\[ f_T(T) = \frac{(10\alpha_w/{\ln 10})}{T^\alpha_g 2\pi \sigma_w^2} \exp \left( -\frac{\left( \frac{c_1 - P_{S-R_1}^{Lg} - 10\alpha_w \ln 10}{T^{\alpha_g}} \right)^2}{2\sigma_w^2} \right), \quad T \geq 0 \quad (23) \]

Thus, \( f_{Lg,S-R_1}(L_{g,S-R_1}) \) can be expressed using multiplication of two random variables [10] i.e., \( \chi_{g, LN} \) and \( T \) as follows:

\[ f_{Lg,S-R_1}(L_{g,S-R_1}) = f_{Lg,S-R_1}(L_{g,S-R_1}) = \frac{1}{c_3^3} \int_{-\infty}^{\infty} f_{\chi_{g, LN}}(t) f_T \left( \frac{L_{g,S-R_1}}{c_3 t} \right) dt, \quad (24) \]

where \( c_3 = 10^8 \beta / 10 \kappa \). By substituting the PDF of \( \chi_{g, LN} \) and (23) into (24), \( f_{Lg,S-R_1}(L_{g,S-R_1}) \) can be finally written as given in (25), which can be solved using numerical integration.

REFERENCES


