Applying Game Theory to Relay Resource Selection in Hybrid-band Wireless Systems

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Abstract—Although emerging relay-based communication networks boast of increased transmission coverage and capacity, they currently lack schemes that optimize the simultaneous utilization of the best-quality channels, especially when such channels belong to heterogeneous frequency bands. In this paper, we present a centralized oracle based on a bipartite graph model for multi-band multi-channel allocation, in this case to relay nodes. We also indicate the reasons for which this technique is not viable for deployment. We then go on to develop a customized greedy heuristic, the traditional choice for distributed multi-band multi-channel assignment to relay nodes, that results in sub-optimal performance. Since the greedy approach does not offer a performance bound guarantee, we also explore a better online, distributed multi-band multi-channel allocation strategy by proposing a sequential algorithm based on game theory. Computer-based simulation results demonstrate that our proposed game-theoretic approach significantly outperforms the traditional distributed and centralized methods with a Nash Equilibrium convergence guarantee.

Index Terms—Multi-band, multi-channel, relay, game theory, sequential game, bipartite graph, optimization.

I. INTRODUCTION

In conventional communications systems relay-based network topology is an established data communication technique, used to improve network’s transmission coverage and capacity [1]. Relaying a data packet from a sender node (SN) via relay node/s (RN(s)) to a destination node (DN) can be advantageous, especially while considering the following two cases: First, if the direct channel between SN and DN does not satisfy the desired quality of service (QoS) and second, if DN is far from the SN. Hence, improving QoS and increasing network coverage but at the cost of a longer delay. In this paper, we introduce the advantages and challenges of using multi-band communication to mitigate this drawback of relay-based communication networks.

With advanced wireless mobile systems, seen in the relay network depicted in Fig. 1, we assume multi-band devices supporting a variety of transceiving frequency bands (e.g., IEEE 802.11ax operating at 2.4GHz and 5GHz [2], millimeter wave (mmWave), visible light communication, and so forth) that react differently to path loss, fading, mobile blocking, and other physical phenomena. As an automatic consequence, there is a large diversity among channel conditions across different frequency bands [3], [4]. Multi-band communication, not only, helps in making mobile devices compatible with different wireless standards but can also be used to reduce the data transmission delay between two nodes.

In this paragraph, first, we discuss the functionality of traditional single-band relay network and then we show the importance of the considered multi-band relay network. Traditionally, in single-band relay network, the RN waits for the SN to complete its transmission, on what we can denote to be the $i^{th}$ channel of the $j^{th}$ band ($c_{ij}^{sn}$). The same channel is then used to forward the data packet to the DN. By functioning in this manner, the packet relaying from SN to RN and then from RN to DN, takes twice the time. This approach is widely known as the decode and forward (DF) scheme [5]. On the other hand, in the considered multi-band relay communication, the RN can start sending data out to the DN using some another $n^{th}$ channel from a different $n^{th}$ band ($c_{mn}^{rn}$), even before the SN-to-RN data transmission gets completed. Naturally, this technique significantly reduces packet forwarding latency, but at the cost of requiring efficient buffer management strategy, which is needed for handling the difference in data rates of the received and re-transmitted data packets, while communicating on multi-bands.

As solutions to the above-mentioned issues of added latency while utilizing relay-based communication and possibility...
of buffer overflow while using multi-band communication, we introduce both centralized and distributed approaches to efficiently assign appropriate channels. As depicted in Fig. 1, when a centralized oracle makes decisions for all the network entities, including RNs, it requires information on the complete network to provide a one-time (non-adaptive) solution. Such a decision-making process requires a non-deterministic amount of time, with a significant increase in the number of nodes. Moreover, such a solution is clearly unsuitable for rapidly-evolving channel environments. This results in a more complex, time-consuming, and static optimization, which is not practically viable for relay networks. Therefore, designing a practical band/channel selection method that minimizes the adverse impact on QoS parameters (e.g., communication delay, throughput, available power/energy, available buffer) in relay-based communication emerges as a key research problem in this paper.

Our desired approach, therefore, hinges upon designing a distributed, online decision-making technique that permits RNs to localize heterogeneous bands/channel selection based on the prevalent channel and traffic conditions. Such a localized distributed decision-making process is naturally lightweight (i.e., less complex), prompt and reliable, thereby improving the overall delay (from SN to DN) and throughput.

**Contributions:** With the objective of achieving reduced data packet latency for multi-band relay network, the contributions of our work, in this paper, are outlined as follows:

1) We provide details on our formerly proposed greedy algorithm, which has natural distributed mechanism, by providing flowchart and detailed packet transfer diagrams.

2) With regards to the centralized oracle approach, we propose a graph theory based solution and model the problem as a bipartite graph [6], which is then solved using tools from optimization theory [7]. Graph theory is ultimately a study of relationships which provides a helpful tool to clarify and evaluate complete dynamics of the problem, hence providing a brief interpretation of a convoluted network.

3) For a distributed approach, with rapid decisions and performance guarantee, we propose a game theory based technique and design the problem as a sequential game with perfect information. Game theory provides a proper framework for understanding choices in the situations having competing players hence making this approach more appealing to explore and study in this scenario. In our proposed sequential game with perfect information (not to be confused with complete information, which was seen in the case of the centralized oracle), the RNs act as players.

4) The RNs elegantly play the proposed sequential game, aiming to derive near-optimal solutions. The game is then represented using a detailed decision tree for rigorous performance analysis.

5) We also provide model-specific computational and theoretical analysis discussing sub-game perfect equilibrium using backward induction for proposed sequential game.

6) The analysis is presented with a simple yet illustrative example, based on which it is deduced that the sequential game can be converged and stabilized by obtaining the Nash Equilibria of the sub-games within the main game.

The remainder of the paper is organized as follows: section II provides a relevant literature review. Our considered centralized oracle based is presented and the formal problem of multi-band multi-channel allocation for RNs is formulated in section III. Then, we present a distributed greedy heuristic to solve this problem in section IV. Next, in section V, a sequential game-theoretic algorithm is presented as an advanced distributed channel allocation method. Simulation results are reported in section VI to compare the performances of the centralized oracle and proposed distributed techniques. Concluding remarks are presented in section VII.

## II. RELATED WORK

Reducing communication delay is an active area of research in relay-based networks. In this section, first, we discuss importance and shortcomings of some of the related research papers [8]–[12], where authors have studied multi-band relay topology with the aim of reducing end-to-end packet latency but fail to achieve the goal. Later we discuss some of the research papers [13], [14], where authors have studied efficient channel allocation algorithms with the aim of increased throughput, and ignored the added latency issue. Then we discuss some of the game and graph theory based channel allocation algorithms [6], [15]–[19], from literature where authors fail to explore the area of multi-band relay networks and their added latency issue.

In [8], a multi-hop network was proposed in which a centralized optimization problem was employed to offer scheduling decisions in terms of routing, channel allocation and link scheduling. This work considers the fact that uncertain channel availability results in uncertain packet scheduling, which not only increases latency but also heavily overloads the system. Such centrally-controlled relay multi-band systems can only be used for smaller networks, where it can be assumed that channel conditions for a specific range would be the same (as sensed by the central node (CN)) and would remain constant for a certain period of time.
A multi-band transmission based relay model is proposed in [9]. This proposed model is for WLAN systems and also aims to reduce packet latency. This work carries out relay transmission and frame reception at the same time by reading the truncated header instead of complete header. To rephrase it, while receiving the data packet, the RN relays (re-transmits) data packet to the next node right after reading the truncated header instead of reading it completely. Such strategy is called as truncated decode and forward scheme. In order to obtain the information for judging whether or not relay transmission is required, RN receives the header of a frame on one frequency band. If relay transmission is required, the RN starts the relay transmission on another frequency channel while the RN continues to receive the frame [9]. However, reading partial header, increases the chances of frame error as, it can degrade the de-coding performance. This might result in packet re-transmission, hence increasing the packet latency.  

In [10], the authors provided practical experimental results for a multi-band WLAN system. This system finds scattered and unused spectral resources across multiple bands and uses them for frame re-transmission. Therefore, utilizing multi-band WLAN can improve the total spectral efficiency when compared to the legacy single-band WLAN. Furthermore, to effectively utilize spectral resources, this algorithm judges, based on the prediction of an idle/busy channel, whether the node should immediately transmit using a single band or wait for another channel to get free, in order to perform multi-band simultaneous transmission and reception. This work does not utilize the relay technology and can therefore only be used for lesser range networks. 

In [11], the authors proposed a lightweight algorithm that considers appropriate channel usage for receiving/sending data based on SINR, with the ultimate aim of avoiding data loss caused by the difference between SN and RN rates. This work adopts spatial re-use by adjusting a threshold of acceptable interference. In this algorithm, the channels are selected such that the receiving and sending rates are maximized while the transmission delay is minimized. In the algorithm, the SINR of each channel is first calculated for SN to RN and RN to DN transmission, and then the modulation and coding scheme is chosen by looking up an appropriate scheme corresponding to that SINR. This work lacks a strategy that handles the case where better SINR channels are unavailable. In [12], a similar approach is proposed but there is no handling of any specific cases where low SNR channels and high SINR channels are available. 

Researchers in [13] presented a multi-channel allocation algorithm based on congestion-aware (CA), in an attempt to reduce the interference between nodes and to extend their longevity. The algorithm determines the degree of congestion across different channels by comparing the transmission capacity of each of their nodes, ultimately selecting the most suitable channel for data transmission. This algorithm reduces the number of channel selection conflicts and improves the throughput of the whole network as much as possible. 

A channel ranking algorithm was proposed in [14], in which all nodes prioritize available channels on the basis of their channel properties. Afterwards, a distributed channel allocation algorithm is implemented such that each node can choose a suitable channel based on both: its residual energy and channel ranks. Finally, the spectrum-sensing and sleep duration are optimized together in order to satisfy energy consumption constraints and increase the normalized throughput, simultaneously. This approach requires no global information or any central coordination, unlike other former approaches. Here, each node can work in an absolute distributed manner based only on its own local knowledge. Furthermore, theoretical analysis and extensive simulations have validated that, upon applying this solution to the IoT network: (i) each node can be allocated to a proper channel based on the residual energy to balance the lifetime; (ii) the network can rapidly converge to a collision-free transmission through each node’s learning ability during the process of the distributed channel allocation; and (iii) the network throughput is further improved via dynamic time slot optimization. We refer to this work as a Congestion-aware Power-aware (PACA) algorithm. Both, CA and PACA algorithms aim to increase throughput of the network and fail to consider the possible added latency. Later we compare our proposed models with these conventional CA and PACA approaches. 

Graph theory, being a widely used representation approach for complex expanded networks, has been studied in [6], [15], [16] for solving typical resource allocation problems of wireless communication networks. In [6], authors have proposed a graph-based resource allocation scheme for side-link broadcast vehicle-to-vehicle communications, where vehicles and spectrum resources are represented by vertices where as, the edges represent the achievable rate. In [15], a weighted conflict graph based model is studied, so as to compute the throughput of each access point for a given allocation. In [16], the problem of network throughput optimization of an intelligent reflecting surface (IRS)-assisted multi-hop network is investigated, in which the phase-shifts of the IRS and the resource allocation of the relays are jointly optimized. Where as, these approaches [6], [15], [16] lack in studying multi-band relay networks for increased latency issues and focus on increasing only the throughput of the network. 

Game theory, being the most preferred technique for handling environments with competing nodes/players, is studied in literature [17]–[19], for many expanded complex wireless networks. In [17], a full-duplex, amplify-and-forward multiple relay network, employing simultaneous wireless information and power transfer receivers, is studied. This work allocates optimal power values in order to increase network throughput, ignoring the added latency issue of a relay network. In [18], a joint access and back-haul resource allocation in a 5G heterogeneous ultra-dense network are proposed in order to maximize the overall system throughput and fails to address the multi-band relay networks for increased latency issues. In [19], the problem of sub-carrier and power allocation in multi-user single-carrier frequency division multiple access (SC-FDMA) wireless networks is addressed, using a three phase game model. This algorithms finds a trade-off between increased network throughput and energy efficiency in order to find its optimal channel/power.
allocations, while ignoring the increased latency issues of the network.

In spite of the literature being enriched with game and graph theory research, it mostly aims for increasing sum rate without focusing on increased data packet latency specifically in the area of multi-band relay network. Our proposed work not only aims to reduce data packet latency but also shows its effect on the effective throughput of the entire network. Hence it can be said that now is a much more opportune time to solve the added packet latency and buffer overflow problem for multi-band relay network using advanced techniques like game and graph theory.

### III. Considered Centralized System Model and Problem Description

This section presents our considered centralized system model based on a balanced bipartite graph. Here, we also formulate the optimization problem. As shown in Fig. 1, the system model assumes one CN, acting as an oracle and having complete information of the network, i.e., spectrum-sensing results [20], SNR estimation [21] of free channels, buffer size, available power, distance between interconnecting SN, RN(s), and DN, and so forth. In addition, the CN is responsible for making decisions and allocating resources such as power and frequency. For ease of reference for the readers, the major notations and symbols used throughout this section and the remainder of the paper are listed in Table I.

#### A. Bipartite Graph-based Oracle

The relay-based network can be represented as a combinational, constrained-weighted bipartite graph matching problem \( C = (G, Q, E) \) consisting of two disassociated sets of vertices \( G \), \( Q \), and a set of edges \( E = G \times Q \) as depicted in Fig. 2. An edge \( X_{gq} \) connects a vertex \( RN_g \in G = \{RN_1, RN_2, ..., RN_G\} \) with a vertex \( (P, f) \in Q = \{(P, f)_1, (P, f)_2, ..., (P, f)_Q\} \) and has an associated weight \( U_{gq} \), where \( (P, f) \) represents a tuple containing both power level and available channel frequency. The objective is to find a matching \( X \subseteq E \) that associates every vertex in \( G \) with a vertex in \( Q \). In a bipartite graph, when the cardinalities of the vertex sets are equal (i.e., \( |G| = |Q| \)), perfect matching can be attained [6].

The vertices \( RN_g \) represent the RNs that belong to the same set \( G \), whereas the vertices \( (P, f)_q \) represent the allocable resources, which are denoted by set \( Q \). In this work, we consider the packet time span \( U_{gq} \) to be the weight of the edge connecting the \( g^{th} \) RN to the \( q^{th} \) \( (P, f) \) pair. Here, \( U_{gq} = F/(B \log_2(1 + SNR_{gq})) \), \( F \) is the packet size in bits, \( B \) is the bandwidth of the resource and \( SNR_{gq} \) is the function of the \( q^{th} \) \( (P, f) \) pair connected to the \( g^{th} \) RN. The goal is to choose the edges/connections that grant minimum weights, i.e., to find \( X_{gq} \in \{0, 1\} \), where 0 means discarding the connection and 1 indicates keeping the connection.

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
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<tbody>
<tr>
<td>( G )</td>
<td>Set of number RNs present in the network.</td>
</tr>
<tr>
<td>( (P, f) )</td>
<td>Pair of specific power level, ( P ), and the specific available channel, ( f ).</td>
</tr>
<tr>
<td>( Q )</td>
<td>Set of number of ( (P, f) ) pairs available.</td>
</tr>
<tr>
<td>( U_{gq} )</td>
<td>Time span of data packet forwarded from ( g^{th} ) RN using ( q^{th} ) ( (P, f) ) pair.</td>
</tr>
<tr>
<td>( BO_{gq} )</td>
<td>Amount of buffer overflow occurred at ( g^{th} ) RN, by using ( q^{th} ) ( (P, f) ) pair.</td>
</tr>
<tr>
<td>( SNR_{gq} )</td>
<td>SNR achieved by ( g^{th} ) RN while using ( q^{th} ) ( (P, f) ) pair.</td>
</tr>
<tr>
<td>( F_{sd} )</td>
<td>Data frame sent from SN to RN in bits.</td>
</tr>
<tr>
<td>( F_{rd} )</td>
<td>Data frame from RN to DN in bits.</td>
</tr>
<tr>
<td>( t_{sr} )</td>
<td>Time at which RN starts receiving ( F_{sr} ).</td>
</tr>
<tr>
<td>( t_{out} )</td>
<td>Time at which RN schedules the data frame for re-transmitting it to DN ( (F_{rd}) ).</td>
</tr>
<tr>
<td>( T_h )</td>
<td>Time span of the header of the packet.</td>
</tr>
<tr>
<td>( T_{sr} )</td>
<td>Time span of ( F_{sr} ).</td>
</tr>
<tr>
<td>( T_{rd} )</td>
<td>Time span of ( F_{rd} ).</td>
</tr>
<tr>
<td>( T_{tot} )</td>
<td>Total time span from the instant at which RN starts receiving data frame from SN ( (F_{sr}) ) till the time instant, DN completely receives the re-transmitted data frame ( (F_{rd}) ) from RN.</td>
</tr>
<tr>
<td>( D_{sr} )</td>
<td>Rate at which SN transmits data frame to RN ( (F_{sr}) ).</td>
</tr>
<tr>
<td>( D_{rd} )</td>
<td>Rate at which RN re-transmits data frame to DN ( (F_{rd}) ).</td>
</tr>
<tr>
<td>( ch_{s}^{m,n} )</td>
<td>( m )-th frequency channel of ( n )-th frequency band used by RN to transmit ( F_{sr} ) to RN.</td>
</tr>
<tr>
<td>( ch_{r}^{m,n} )</td>
<td>( m )-th frequency channel of ( n )-th frequency band used by RN to re-transmit ( F_{rd} ) to DN.</td>
</tr>
<tr>
<td>( SNR_{sr} )</td>
<td>Estimated SNR of the channel used by SN to transmit ( F_{sr} ) to RN.</td>
</tr>
<tr>
<td>( SNR_{rd} )</td>
<td>Estimated SNR of the channel used by RN to re-transmit ( F_{rd} ) to DN.</td>
</tr>
<tr>
<td>( buffsize )</td>
<td>Buffer size available to the node.</td>
</tr>
<tr>
<td>( f_{rd} )</td>
<td>Frequency channel used by the RN to forward data packet to its next destined node.</td>
</tr>
<tr>
<td>( f_{sr} )</td>
<td>Frequency channel used by the source node to transmit data packet to the RN.</td>
</tr>
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Fig. 2: Bipartite graph for the centralized oracle.
B. Problem Formulation

To describe our considered problem, let $G$ contain the actual number of RNs (i.e., RNs) plus dummy RNs in order to construct a balanced bipartite problem [6]. In addition, let $Q$ denote the number of power level pairs and available free channels. Let $U$, $T$, $SNR$, and $BO$ be $G \times Q$ matrices, where $U = T$. It contains the outcomes of all the edges and is equal to the time span of the forwarded data packet against each power frequency pair. $BO$ is the buffer overflow matrix against each edge, and $SNR$ denotes the corresponding SNR for each pair. Let $P$ and $f$ be the row vectors of size $Q$, representing the corresponding power values and available channel frequencies for all the pairs, respectively.

Now, our considered multi-band multi-channel allocation problem can be formally modeled as an optimization problem as follows:

\[
\min \sum_{g=1}^{G} \sum_{q=1}^{Q} X_{g,q} \cdot U_{g,q}, \quad (1a)
\]

subject to

\[
\sum_{g=1}^{G} \sum_{q=1}^{Q} X_{g,q} \cdot BO_{g,q} = 0, \quad (1b)
\]

\[
\sum_{q=1}^{Q} X_{g,q} \cdot P_q \leq P_{\text{avoid}}, \quad \forall g, \quad (1c)
\]

\[
X_{g,q} + X_{g+1,q} \leq 1, \quad \forall q, g, \quad (1d)
\]

\[
\sum_{q=1}^{Q} X_{g,q} \cdot SNR_{g,q} \geq SNR_{\text{min}}, \quad \forall g, \quad (1e)
\]

\[
\sum_{q=1}^{Q} X_{g,q} = 1, \quad \forall g, \quad (1f)
\]

\[
X_{g,q} \in \{0, 1\}, \quad (1g)
\]

where $X_{g,q}$ is a $G \times Q$ matrix, of binary decision variables, which are unknown and is to be found.

The objective function (1a) aims to minimize the sum of all the forwarded packets’ time spans. The equality constraint (1b) checks if all the RNs have a buffer overflow equal to zero. The inequality constraint (1c) aims to keep the power usage in accordance with the available power of the RNs. On the other hand, the inequality constraint (1d) forces the system to use a multi-band/channel setting. Constraint (1e) maintains the QoS of each forwarded data packet. The equality constraint (1f) compels each RN to pick one power frequency pair to forward the data packet. Constraint (1g) is for characterizing the unknown matrix $X$ as a Boolean variable.

C. Oracle-based multi-band multi-channel allocation to relays

The steps of the aforementioned centralized bipartite graph-based oracle are enumerated in Algorithm 1, where we minimize the objective function (1a). First, the CN accesses its central spectrum-sensing results and generates $(P, f)$ pairs with all possible combinations of power levels and available channel frequencies. After generating these pairs, the CN collects information on the packet size $F$ directly from the SN (using any information channel). Then, CN collects information on distances between each RN and the power available for each. Next, the CN generates matrices for time span, buffer overflow, and SNR against each power frequency pair for each RN.

After this, the CN generates the equality and inequality matrices for an optimization solver (eq. 1). The solver returns the solution matrix $(X_{g,q})$, which then gets transmitted to every RN, along with packet re-transmitting times for each RN, $tor,g$ (Algorithm 1). Now, the SN commences transmitting, at which point each RN already knows which power frequency is to be selected in order to forward the received data packet to the DN.

While this centralized optimization algorithm requires an oracle (i.e., an entity with full knowledge of the system), it is not progressive with respect to the individual RNs. Due which, in oracle approach, it is not possible to incorporate the ability of accessing local (at RNs) buffer overflow conditions while performing packet relaying. As, the buffer overflow conditions depend on prior nodes’ channel decisions [22] and in centralized approach, these decisions are taken all at once instead of being decided individually. In other words, in the centralized model, all decisions pertaining to the band and channel allocation are taken at once by the CN. Therefore, the oracle can only check the buffer overflow condition for all the nodes simultaneously and not individually.

With an increasing number of RNs, however, this approach takes non-deterministic time and adds exponential latency to the system. This type of approach would only be suitable for devices with unlimited power, where it is capable of finding the global optimal solution for a network with infinite buffer sizes. Due to such impracticality, a distributed solution would be a better strategy when attempting to solve this problem.

IV. CUSTOMIZED GREEDY HEURISTIC FOR DISTRIBUTED MULTI-BAND MULTI-CHANNEL ALLOCATION

In the conventional DF scheme, RN has to wait for the receiving channel to become available so that it can forward the data packet to its next node. This, inevitably, increases packet transmission latency, as shown in the packet timing diagram of DF in Fig. 3. In [22], we modeled and solved a multi-band relay communication model using a distributed greedy algorithm to reduce the added latency of a relay communication network. We extend that concept in this paper, and we develop a customized greedy heuristic in order to handle the two possible cases that can be faced by a RN, when forwarding data packet to the next node/destination: availability of higher or lower SNR channel.

Once SN transmits data packet to a RN, at a specific data rate, the RN, using multi-band communication, can forward this data packet to the next node/destination utilizing a different channel of an alternate band (hence using a different data rate). This difference in data rates requires an available buffer to store the data. As can be noticed from the flowchart in Fig. 4, upon receiving the data packet from the SN at $SNR_{sr}$ and $D_{sr}$, the RN checks if it has greater

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SNR channels available in the same or in a different band. If yes, then RN finds the highest SNR channel or adjusts the data rate on the highest SNR channel in order to meet buffer overflow condition. Once the buffer overflow condition is met, RN schedules the data packet out at \( t_{or} \). If higher SNR channels are not available then RN finds the best SNR channel among the available channels or accesses its updated channel information in order to meet buffer overflow condition. Once the buffer overflow condition is met, RN schedules the data packet out at \( t_{or} \).

A detailed timing diagram for the case of a lower SNR channel is shown in Fig. 6. It can be seen from the figure that if the RN receives the packet at a higher data rate from the prior node, the RN has to transmit the packet using a lower SNR channel according to the availability of bands/channels.
on one time greedy decisions which directs the algorithm to quickly choose the best feasible channel and schedule the data packet for re-transmission at \( t_{ofr} \). This approach is handled with the least complexity, where as there is no proper structure for in depth analysing of the solutions as a network.

Our customized greedy heuristic can be used as a reference since it provides the first intuitive and distributed solution to the optimization problem. However, due to its greedy nature, there is room for further improvement with regards to the quality of the solutions, using a proper framework. We will investigate this using a sequential game theoretic approach in the following section.

V. PROPOSED GAME THEORETIC DISTRIBUTED MULTI-BAND, MULTI-CHANNEL SELECTION ALGORITHM

In this section, we model our proposed multi-band multi-channel relay communication system (Fig. 1) as a sequential game with perfect information. We first present the motivation behind adopting the sequential game and its necessary preliminaries, which is then followed by our envisioned game representation, problem reformulation, detailed theoretical analysis and an illustrative example.

A. Sequential Game Model: Motivation and Preliminaries

A sequential game involves a model in which one player performs an action (distributively) before other players make their decisions (i.e., no two players can make a move at the same time) [23]. In our case, a particular RN can start transmitting at a particular power and channel (i.e., it makes a decision) only after it receives the packet from the previous node. Importantly, the later players must have some information about the earlier choices made; otherwise, the difference in time would have no strategic effect. In our case, the packet header received from the previous node provides this information to the particular RN.

Perfect information is often confused and used interchangeably with complete information; however, here we make an important distinction. In this model, we assume that RNs receive perfect information in the packet header from the prior node. In our case, we do not use complete information.

Fig. 6: Packet timing diagram for the case where only lower SNR channels are available.

1) Players: In our case, RNs are regarded as players.
2) The information available to each player: Here, RNs hold information about the packet size/length as well as about the rate at which data is received.
3) Possible moves: Actions available to every RN which, include selecting a power frequency pair out of \(\mathcal{P}\) possible power values \( P = [P_1, P_2, ..., P_T] \) and \(\mathcal{F}\) available channels \( f = [f_1, f_2, ..., f_F] \), at which the data can be forwarded to the next player.
4) The payoffs for each outcome: Here, the payoff is a Boolean representation. Either all the constraints are met against each power and frequency channel pair, or they are not.

In game theory, these elements are typically used along with the strategies available to each player. Here, each RN’s strategy is to choose a power frequency pair that satisfies the utility function and fulfills all constraints. The utility function of the modeled game is to reduce the forwarded packet’s time span.

The sequential aspects, characterized by the extensive form representation can also be depicted as a decision tree, which can be seen in Fig. 7. The decision tree demonstrates the possible ways of playing our considered game. \( D_{sr1} \) denotes the initial data rate chosen by the source/SN in accordance with its own channel condition. To trigger the relay communication, this value is considered the starting/initial point for the necessary algorithm to be designed.

B. Reformulation of the Original Problem

Now, we transform the original optimization problem in eq. (1a) into the distributed game model for \(g^{th}\) player (RN) as follows:

\[
\min U = \min T_{g,(g+1)}(P_{g,(g+1)}, f_{g,(g+1)}),
\]

Subject to (\(po \in \{0,1\}\))

\[
BO(\mathcal{D}_{(g-1)}, \text{bufsize}_g) = 0, \quad f_{(g-1),g} \neq f_{g,(g+1)}
\]

\[
\text{SNR}_{g,(g+1)} \geq \text{SNR}_{\text{min}}
\]

\[
0 \leq P_{g,(g+1)} \leq P_{g,\text{avail}},
\]
These are used to transmit data packets from the distributively and on the basis of current/updated local limited number of branches. This approach makes decisions of movable devices.

The constraint (4e), however, is the power limited constraint representing 1 for all constraints met and 0 otherwise. Constraint (4c) is defined as the respective QoS constraint that maintains the data quality.

and $P_{g,(g+1)}, f_{g,(g+1)}, T_{g,(g+1)}$ and $d_{g,(g+1)}$ are the chosen power, channel, data time span and distance respectively. These are used to transmit data packets from the $g^{th}$ RN to the next $(g + 1)^{th}$ node.

The constraint (4b) states that while using the particular $(P, f)$ pair, the buffer should not overflow, and this is checked against the $(g - 1)^{th}$ RN’s data rate ($D_{(g-1)}$) and the $g^{th}$ RN’s available buffer size ($\text{bufsize}$). Constraint (4c) is defined as forcing the player to use the multi-channel/multi-band opportunity in order to reduce latency. Constraint (4d) is the respective QoS constraint that maintains the data quality. Constraint (4e), however, is the power limited constraint of movable devices. $po$ is the payoff Boolean variable representing 1 for all constraints met and 0 otherwise.

C. Envisioned Game Theoretic Solution

Now, we propose Algorithms 2-3 to progressively minimize the utility function (5) at the RNs. The steps of the algorithm are explained below:

1) The SN transmits data of size $F$ (bits) to the RN$_{1}$.
2) The player RN$_{1}$ extracts the perfect information, including the incoming data’s size $F$ and rate $D$ from the header of the received packet.
3) The player RN$_{1}$ chooses the power value and channel $(P, f)$ pairs out of the $\mathcal{P}$ possible power values $P = [P_{1}, P_{2}, ..., P_T]$ and $\mathcal{F}$ available channels $f = [f_1, f_2, ..., f_{F}]$, and calculates the corresponding outcome of the utility function ($U$) as well as the payoff ($po$) for each pair.
4) The utility function $U$ is the corresponding time span, and $T$ of each packet is to be forwarded on the basis of a particular $(P, f)$. The payoff, $po$ is a Boolean variable indicating 1 for the successful fulfillment of all constraints and 0 otherwise.
5) The player RN$_{1}$ then selects one pair out of all others, resulting in $po = 1$ and a minimum $U$ among them. Therefore, it selects the pair with the minimum $U$, which fulfills all the constraints.
6) Since this is a sequential game, each player (RN) takes turns in a linear, progressive fashion. When the first RN finalizes its selection of $(P, f)$, it can forward packets to the next node, providing information about $F$ and $D$ in the header. Now, the next node is ready to play the game as was done by RN$_{1}$.

The proposed algorithm is simple and efficient for a limited number of branches. This approach makes decisions distributively and on the basis of current/updated local band/channel conditions. It is hence capable of producing more reliable results. Moreover, all constraints can be handled by this approach, rendering our game theoretic algorithm suitable for both limited power and buffer size.

D. Computational/Theoretical Analysis

In this section, a brief computational and theoretical analysis of the proposed game theoretical model is provided. We discuss the idea of equilibrium for extensive games with perfect information, particular in the context of wireless communication networks.

In game theory, in order to find the best strategy for each player, the theorem of Nash Equilibrium is used [25]. Nash Equilibrium is an ideology where each device (player) has some or all of the information of the corresponding network and there are no other parameter adjustments (moves/strategy) left that can enhance its performance (utility). To choose its next strategy, each player first looks for the network (other devices) information available to it and then opts for the strategy that can enhance its performance (a step forward).

In other words, it can be said that each player chooses a strategy from which it will never deviate (as any other strategy will not enhance its performance) so long as it cannot go a step higher. Nash Equilibrium is also said to be the most "dominant strategy" leading towards better results out of all other possible moves. Nash strategy is not always the most optimal strategy but is actually the best one.

For sequential games with perfect information (also known as "extensive games", "strategic games" and "dynamic games"), in order to find the best strategy for each player, sub-game perfect equilibrium (SPE) is found using backward induction [26], [27]. This is the process of studying and analyzing the results of a sequential game played in a forward trend: where the decision-making process starts from the first node and ends at the last one. Now all decision-takers are fully aware of the actions chosen by the previous and next players.

Backward induction, on the other hand, occurs when the decision-making process starts from the last node and ends at...

Input: $(P, f), T_{(g-1),g}, f_{(g-1),g}, t_{ir}$
Output: $U = T_{g,(g+1)}, D_{g,(g+1)}, (P, f)_{g,(g+1)}$

for $q = 1 : size(P, f)$ do

1. $BC = PC = SC = FC = zeros(1,Q)$;
2. Calculate $T(q), D(q),$ and $SNR(q)$ using eqs. (5), (6), and (7), respectively;
3. Check buffer overflow constraint using eq. (4b):
   if $T(q) < T_{(g-1),g}$ then
     if $(T(q) - T_{(g-1),g}) * D_{(g-1),g} \leq \text{buffersize}$ then
       $BC(q) = 1$;
     else
       $BC(q) = 1$;
   4. Check multiband/channel constraint using eq. (4c):
     if $f(q) \neq f_{(g-1),g}$ then
       $SC(q) = 1$;
     5. Check QoS constraint using eq. (4d):
       if $SNR(q) > SNR_{min}$ then
         $SC(q) = 1$;
       6. Check limited power constraint using eq. (4e):
          if $P(q) > P_{g,avail}$ then
            $PC(q) = 1$;
          if $BC(q) = 1 \& FC(q) = 1 \& SC(q) = 1 \& PC(q) = 1$ then
            $po = 1$;
            $indices = \text{find}(po = 1)$;
            $T_{g,(g+1)} = \text{min}(T(indices))$;
            $index = \text{find}(T == T_{g,(g+1)})$;
            $P_{g,(g+1)} = P(index)$;
            $f_{g,(g+1)} = f(index)$;
            $SNR_{g,(g+1)} = SNR(index)$;
return $T_{g,(g+1)}$

the first one, which is then used to accomplish equilibrium. For backward induction, first, the game is sectioned into sub-games, as per the rules [28]. Then starting from the last player, the best action for last player is chosen. Then the next-to-last player chooses its best move (this time having the knowledge of moves available to the last player) in order to increase the overall utility. This process is explained below using an illustrative example involving a relay network:

E. Illustrative Example

Here we consider the sub-game played between $(g - 1)^{th}$ and $g^{th}$ RNs, while performing backward induction with the aid of Fig. 8. As discussed earlier, each branch has its corresponding payoff and utility functions, and we assume that initially, during the forward sequential game, the branches with $po = 0$ are already discarded. Starting in a backward fashion from $g^{th}$ RN. First, we find (not select) the branches having the minimum utility i.e.:

![Fig. 8: Sub-game perfect equilibrium between $g^{th}$ and $(g - 1)^{th}$ RN during backward induction.](image)

$X_g^1 = \min\{U_g^1(U_g^1)_{g-1}, U_g^2(U_g^2)_{g-1}, U_g^3(U_g^3)_{g-1}\}$, (9)

$X_g^2 = \min\{U_g^1(U_g^1)_{g-2}, U_g^2(U_g^2)_{g-2}, U_g^3(U_g^3)_{g-2}\}$, (10)

$X_g^3 = \min\{U_g^1(U_g^1)_{g-3}, U_g^2(U_g^2)_{g-3}, U_g^3(U_g^3)_{g-3}\}$. (11)

Next, the $(g - 1)^{th}$ RN is going to select its move knowing $X_g^1, X_g^2$ and $X_g^3$. In SPE, the most dominating strategy for the $(g - 1)^{th}$ RN is to choose a branch that improves the total utility of this sub-game i.e.:

$X_{g-1} = \min\{(U_g^1 + X_g^1), (U_g^2 + X_g^2), (U_g^3 + X_g^3)\}$, (12)

where, $X_{g-1}$ is the SPE of the sub-game played between the $(g - 1)^{th}$ and $g^{th}$ RNs. Similarly, this $X_{g-1}$ can be used by the $(g - 2)^{th}$ RN in order to find its SPE, $X_{g-2}$, for the sub-game played between the $(g - 2)^{th}$ and $(g - 1)^{th}$ RNs. The same procedure is followed by each RN until the backward induction reaches the first RN that is $g = 1$. In section VI, we present example of backward induction from out simulations.

F. Proof: A sequential game converges to Nash Equilibrium with SPE using backward induction

In game theory, Nash Equilibrium is a state of stability where there is no better strategy left for any player to play. In a sequential game, which has to be played in the forward direction (where, the players, do not have all the information of future/next players available), the players cannot reach a state of Nash Equilibrium during the forward iteration. The sequential game can then be analyzed and converged to its Nash Equilibrium by finding Nash equilibria of its sub-games (each having at least two players) using backward induction i.e., iterating backwards in the game. This time, the
prior players are enlightened with future information that was previously unavailable in the forward iteration.

In other words, with reference to the illustrative section above (V-E), $X_{g-1}$ is the Nash Equilibrium of the sub-game played between $(g-1)^{th}$ and $g^{th}$ RN. Moving backward, $X_{g-1}$ is used in finding the Nash equilibria of the sub-game played between $(g-2)^{th}$ and $(g-1)^{th}$ RNs, $X_{g-2}$, eventually converging towards the Nash Equilibrium of the overall game. In conclusion, it can be said that Nash Equilibrium of a sequential game is equal to the overall Nash of its sub-games.

VI. PERFORMANCE EVALUATION

In this section, we evaluate the performance of our proposed game theoretic approach using computer-based simulations. We also compare it with the centralized oracle, distributed greedy heuristic, CA, PACA, and random channel allocation algorithms. MATLAB is used to simulate the near-to-real time behaviour of the algorithms, and to generate the simulation plots, while keeping the algorithms’ execution times into consideration. Following parameters are used in simulations: $F = 3000$ (bits), $SNR_{min} = -5$dB and $BW = 20$MHz.

With reference to sections V-E and V-F, Tables III, IV, V and VI demonstrate simulation results for two sample scenarios where Q=9 and G=3. The simulation parameters for these examples are shown in Table II, where, $g = 1, 2, 3$. Tables III and V list the values for $po$ for $RN_1$ and the corresponding $po$ of $RN_2$ on the active ($po = 1$) $RN_1$ branches. Moreover, these tables also represent the corresponding $X_1$ for $RN_3$ and $U_2$ for $RN_2$.

Tables IV and VI exhibit the results achieved using backward induction, where $X_2$ is said to be the Nash Equilibrium of the sub-game between $RN_2$ and $RN_3$ and $X_1$ is the Nash of the Nash for the sub-game between $RN_1$ and $RN_2$. Example 1 (Table IV) shows results for the case where backward induction converges to choose better branches as compared with the forward game iteration. Example 2 (Table VI) illustrates the case where backward induction results in choosing the same branches as the ones picked during the forward game iteration.

In Fig. 9, the proposed greedy, oracle, and game-theoretic approaches are compared with random channel assignment and the conventional CA and PACA approaches for $Q = 100$, $50 < d_g(m) < 1000$, $0.05 < P_{avail}(W) < 0.5$, $100 < \text{buffersize}_g(\text{bits}) > 300$, $0.001 < P(W) < 0.5$. It can be seen from the referred figure that in random channel allocation, the RNs select channels randomly (without any strategy), leading them to take the longest time to relay the data packet. Furthermore, it can be noticed that PACA algorithm performs better than many other approaches but still has poor performance as compared with the proposed game model. The

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values (Example1)</th>
<th>Values (Example2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{r1}$</td>
<td>172 Mbps</td>
<td>91.2 Mbps</td>
</tr>
<tr>
<td>$f_{r1}$</td>
<td>4.87 GHz</td>
<td>4.87 GHz</td>
</tr>
<tr>
<td>$P(f)$</td>
<td>0.001, 0.25, 0.5 (W), 1.2 G, 850 M, 1.68 G (Hz), 860 M, 2.75 G, 5.25 G (Hz)</td>
<td>0.001, 0.25, 0.5 (W), 1.2 G, 850 M, 1.68 G (Hz), 860 M, 2.75 G, 5.25 G (Hz)</td>
</tr>
<tr>
<td>$P_{avail}$</td>
<td>0.3, 0.2, 0.05 (W)</td>
<td>0.2, 0.3, 0.1 (W)</td>
</tr>
<tr>
<td>$\text{buffersize}_g$</td>
<td>2383, 2042, 2251 (bits)</td>
<td>2476, 2315, 2239 (bits)</td>
</tr>
<tr>
<td>$d_g$</td>
<td>477, 359, 165 (m)</td>
<td>413, 211, 193 (m)</td>
</tr>
</tbody>
</table>

| TABLE III: Backward induction Example 1. |
|---|---|---|---|---|---|---|---|---|---|
| $po$ (RN$_1$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| $U_2$ | 9.39e-05 | 0 | 0 | 1.59e-05 | 0 | 0 | 1.65e-05 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $X_3$ | 6.9e-05 | 1.5e-05 | 1.59e-05 | 6.58e-05 | 0 | 0 | 0 | 0 | 0 |
| 2.19e-04 | 6.24e-05 | 9.15e-05 | 6.58e-05 | 0 | 0 | 0 | 0 | 0 |
| 9.73e-05 | 1.48e-05 | 1.65e-05 | 6.58e-05 | 0 | 0 | 0 | 0 | 0 |
| 9.81e-05 | 1.62e-05 | 1.59e-05 | 6.58e-05 | 0 | 0 | 0 | 0 | 0 |

| TABLE IV: Example 1: Backward induction converging to better results than prior forward iteration. |
|---|---|---|---|---|---|---|---|---|---|
| $X_1$ | $U_2$ | $X_2$ | $X_3$ | $X_4$ |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2.14e-04 | 6.60e-05 | 7.99e-05 | 6.79e-05 |
| Backward Induction | 9.69e-05 | 1.5e-05 | 1.59e-05 | 6.58e-05 |
| 2.19e-04 | 6.24e-05 | 9.15e-05 | 6.58e-05 |
| Forward Game | 9.73e-05 | 1.48e-05 | 1.65e-05 | 6.58e-05 |
| 9.81e-05 | 1.62e-05 | 1.59e-05 | 6.58e-05 |

| TABLE V: Backward induction Example 2. |
|---|---|---|---|---|---|---|---|---|---|
| $po$ (RN$_1$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $U_2$ | 0 | 0 | 0 | 6.45e-05 | 0 | 0 | 1.4e-04 | 0 | 0 |
| 0 | 0 | 0 | 6.45e-05 | 0 | 0 | 0 | 0 | 0 |
| 3.92e-05 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3.92e-05 | 0 | 0 | 6.45e-05 | 0 | 0 | 0 | 0 | 0 |
| 3.92e-05 | 0 | 0 | 6.45e-05 | 0 | 0 | 0 | 0 | 0 |
| $X_3$ | 0 | 0 | 0 | 9.03e-05 | 0 | 0 | 2.67e-04 | 0 | 0 |
| 0 | 0 | 0 | 9.03e-05 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 9.03e-05 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 9.03e-05 | 0 | 0 | 0 | 0 | 0 |
random, CA and PACA ones, for greedy, game, and oracle approaches as compared with the approaches. It is still superior in performance to the random and oracle approaches. The CA algorithm performs worse than most approaches; however, it is still superior in performance to the random and oracle approaches.

In Fig. 10 we provide the buffer overflow results of the greedy, game, and oracle approaches as compared with the random, CA and PACA ones, for $G = 5$, $Q = 20$, $50 < d_g(m) < 100$, $1.5 < P_{\text{avail}}(W) < 3$, $\text{buffsize}_g(\text{bits}) = 100, 200, 300$, $0.01 < P(W) < 3$. It can be seen from the figure that even in the worst-case scenario, the proposed game model still provides a minimum number of buffer overflow bits as compared with all other approaches. It can also be noticed that the oracle approach has a smaller buffer overflow than all other approaches save for that of the proposed game model. However, the PACA and CA algorithms lie in the middle for this criteria.

Because the greedy approach ignores the power usage consideration, it ends up depleting all the power available for that particular transmission, as evident from the results in Fig. 11 (where, $G = 8$, $Q = 100$, $50 < d_g(m) < 1000$, $2 < P_{\text{avail}}(W) < 3$, $\text{buffsize}_g(\text{bits}) = 100, 0.01 < P(W) < 3$). When solving the problem centrally, the oracle-based approach considers the buffer size to be the maximum, whereas in our game theoretic, the distributed approach remains tightly constrained with the buffer overflow condition.

As a consequence, our proposal uses the least amount of power among all the methods.

The effective throughput is defined as the number of successful packets received during total time of relaying. It can be seen from Fig. 12 (where, $Q = 100$, $50 < d_g(m) < 1000$, $0.05 < P_{\text{avail}}(W) < 1$, $\text{buffsize}_g(\text{bits}) = 1000$, $0.001 < P(W) < 0.5$) that the effective throughput of the proposed game theoretic method is the highest, maximizing time efficiency while minimizing buffer overflow. On the other hand, the oracle-based method has the least effective throughput, demonstrating an exponential rise in packet latency.

In Fig. 13 we provide the effective throughput of the
approaches for $G = 4$, $Q = 100$, $50 < d_{\text{g}}(m) < 1000$, $0.05 < P_{\text{avail}}(W) < 0.5$, $100 < \text{buffsize}_{(\text{bits})} < 3000$, $0.001 < P(W) < 0.5$. It can be seen from the figure that even in the worst-case scenario, the proposed game model has the highest effective throughput, whereas, the random channel allocation has the least effective one because of its random strategy selection. The CA algorithm performs better than oracle and random ones but still has a smaller effective throughput than that of the game, greedy, and PACA models. The PACA algorithm performs better than other approaches but still has smaller effective throughput as compared with the proposed game model.

VII. CONCLUSION AND FUTURE WORK

In this paper, we addressed the need to efficiently assign channels from heterogeneous frequency bands to RNs with the aim of improving the overall transmission time and effective throughput without an increase in the energy consumption of the RNs. First, we present the centralized approach using a bipartite graph. Due to the practical limitation of a centralized oracle, we considered a greedy heuristic to provide a sub-optimal performance. Then, we developed a distributed sequential algorithm using game theory. Later we provide the model-specific Nash Equilibrium analysis for the proposed game model. The results from simulations demonstrate that the performance of our proposed model eclipses that of traditionally used approaches, including a centralized oracle and a greedy heuristic-based distributed method.

Moving on-wards, we are working on proposing algorithms with smart strategy for addressing the environment dynamics like mobility and changing channel situations. To be precise we are working on designing artificially intelligent radios capable of performing reinforcement learning to comprehend the erratic dynamics of the environment and take the optimal decisions accordingly.

REFERENCES


