III Interactions of \( e^+ e^- \) with matter

C) Compton scattering

Compton scattering is like the photo-electric effect except the photon isn't absorbed but deflected by atomic electrons.

Ideal Compton scattering is described in terms of free electrons.

**Before**

\[ \text{before} \]

\[ \text{\( e^- \)} \]

**After**

\[ \text{\( e^- \)} \]

\[ \text{\( \phi \)} \]

\[ \text{\( E_n, \lambda_n \)} \]

The collision is elastic.

\[ \lambda' = \lambda + \lambda_c (1 - \cos \Theta) = \frac{2\pi e}{\lambda} = \frac{c}{\lambda} = \frac{12700}{E_{\gamma}} \]

\[ \lambda_c = \frac{h}{mc} = 2.43 \times 10^{-12} \text{ m} \]

Electric Compton wavelength

\[ E_n = \frac{h \lambda_c (1 - \cos \Theta)}{\lambda} \]

\[ = \frac{h}{\lambda} \]

\[ \text{Final kinetic energy} \]

\[ \phi = \cot \left[ \left( 1 + \frac{\lambda_c}{\lambda} \right) \tan \left( \frac{\Theta}{2} \right) \right] \]

**Note**: 1) \( \phi_{max} = \frac{\pi}{2} \Rightarrow \text{no electrons can be} \]

\[ \text{backscattered for the Compton process} \]
III Interactions of \( e^+ e^- \) with Matter

2) Compton Scattering.

Note: \( \theta = \pi \Rightarrow E_{\text{max}} \) energy transferred to \( e^- \)

\[
E_{\text{max}} = \frac{2 h \nu}{\lambda_c} \left( \frac{1}{\lambda + 2 \lambda_c} \right)
\]

Ref: Weid the "Compton Edge"

\[
E_{\text{max}} = \frac{2 E_0}{\lambda + 1}
\]

\[
\lambda_c = \frac{c}{2 \pi \nu} = \frac{c}{2 \pi \nu} \approx 2.43 \times 10^{-12} \text{ m}
\]

\[
E_\gamma = \frac{E_0}{c^2} + \frac{E_0}{c^2} + \frac{E_0}{c^2} = \frac{E_0}{c^2} \left( 1 + \frac{1}{4 \times 10^{-6} E_\gamma} \right)
\]

\[
E_\gamma = \frac{4 \times 10^{-6} E_0}{1 + 4 \times 10^{-6} E_\gamma}
\]

\[
E_\gamma = 5 \text{ MeV} \Rightarrow \frac{E_\gamma}{E_{\text{max}}} \approx 250 \text{ eV} = \text{max energy lost by an incident photon}
\]

Energy distribution of Compton recoil electrons

[Graph showing energy distribution with peaks at \( E_0 = 1.5 \text{ MeV} \) and \( E_0 = 10 \text{ MeV} \)]
## III. Interactions of e⁻ + δ, with matter

### a) Compton Scattering

Cross-section: The Klein-Nishina Formula

\[
\sigma = \frac{\hbar \nu}{m_e c^2} = \frac{E_r}{E_0} = \frac{E_r}{0.511 \text{ MeV}} \approx 2 \left( \frac{E_r}{\text{MeV}} \right)^2
\]

The differential cross-section is

\[
\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \frac{1 + \cos^2 \theta + \frac{\xi^2 (1-\cos \theta)}{1 + \xi (1-\cos \theta)}}{\left[1 + \xi (1-\cos \theta)^2\right]^2}
\]

Integrating over \( d\Omega \)

\[
\sigma_{\text{compt}} = \int d\Omega \frac{d\sigma}{d\Omega} = 2 \pi r_e^2 \left\{ \frac{1 + \frac{3}{\xi^2}}{2(1+\xi)} - \frac{3}{2} \ln(1+2\xi) \ln \left( \frac{2(1+\xi)}{1+2\xi} \right) \right\}
\]

\[+ \frac{1}{2(1+\xi)} \ln(1+2\xi) - \frac{1+3\xi}{(1+2\xi)^2} \]

### Diagram

Graph showing \( \sigma_{\text{compt}} \) (in barns) as a function of \( E_0 \) (in MeV). The graph shows a decrease in cross-section with increasing energy.
III Interaction of $e^- \gamma$ with matter

2) Compton scattering

Energy distribution:

$$\frac{d\sigma}{dE_e} = \frac{\pi \gamma^2}{m_e^2 s^2} \left[ 2 + \frac{s^2}{s(1-s)^2} + \frac{s^2}{1-s} \left( \frac{s-2}{s} \right) \right]$$

$$s = E_{\gamma} \frac{E_e}{m_e^2}$$

$$J = \frac{E_\gamma}{m_e^2} = 2 \frac{E_{\gamma}}{m_e^2}$$

$$[\sigma_{\text{elastic}}] = 0.0794 \text{ barns} \quad [\gamma m_e^2] = 0.511 \text{ MeV}$$

**GEANT4:** GEANT4 parame-trized to compute cross-section to reproduce the data down to 10 keV using the equation:

$$T(E, E_{\gamma}) = \left\{ \begin{array}{ll}
\frac{\sigma(E_{\gamma}) \log(1 + \gamma)}{s} & 1 \\
\frac{1 + \gamma^2}{1 + \gamma^2} & \end{array} \right.$$

$$\sigma(E_{\gamma}) = \frac{2 \gamma}{E_{\gamma}^2} \left( E_{\gamma}^2 + E_\gamma^2 \right)$$

$$1 \leq \gamma \leq 100 \quad \text{where} \ E_\gamma \leq 100 \text{ GeV}$$

Source of data for fit:


III Interaction of e+ with matter

(c) Compton Scattering.

GEANT 4:

In addition to the default and low energy models which come with GEANT 4, there is also a model called "GYLECS" which may be installed.

a) G4 Compton Scattering: listed as "compt" in process name. No Rayleigh scattering in code.

b) G4 Low Energy Compton: listed as "comptg" in process name. It's a "Low En Compton" in tracking code. An error is indicated at Rayleigh scattering and no Doppler broadening (effect of bound electron momentum on scattered particle energies).

c) GYLECS: bound electron scattered corrected as "shell-by-shell" basis. Rayleigh scattering from coherent scattering k-scal; form factor data. Doppler broadening included (the result of complex telescope simulation work).

Note: Thomson + Rayleigh scattering are classical processes related to Compton scattering. Klein-Nishina formula reduces to Thomson k-scal if low E and $\theta_{max} = \frac{\theta}{2}$.

Rayleigh scattering is photon scattering from a low or whole coherent scattering. No energy is transferred to medium in either case. Only a