II. Stopping Power

b) Range Straggling

Def. of Range: = Distance traveled before all energy is lost.

\[ R = \int_0^T \frac{dE}{(dE/dx)} \]

Note units: \( dE \) in MeV, \( (dE/dx) \) in MeV/cm, energy \( T \)

Because of energy straggling, though the value of \( R \) can fluctuate, this is called Range Straggling.

If we do an actual experiment where we have a source of particles of a source of incident particles of energy \( E \) and we count how many punch through a material of thickness \( x \), we would see a transmission coefficient \( n(\theta) \) which looks like:

\[ \text{Graph showing transmission coefficient varying with } x \]
II. Stopping Power

(a) Range Straggling

\[ \text{Fractional Range Straggling} = \frac{\nabla R}{R} \]

Assuming the energy loss of a non-relativistic heavy ion through matter follows a Gaussian (thick absorber), it can be shown that

\[ \frac{\nabla R}{R} \approx \frac{1}{2} \sqrt{\frac{M}{A \mu_n}} \]

where

- \( M = \) mass of ions
- \( A = \) atomic \# of projectile
- \( \mu_n = \) total electrons

\[ m_i = 1 \text{amu} = 1.66 \times 10^{-27} \text{kg} \]
\[ m_e = 9.11 \times 10^{-31} \text{kg} \]

\[ \frac{\nabla R}{R} \approx \frac{1}{2} \sqrt{\frac{9.11 \times 10^{-31} \text{kg}}{A (166 \times 10^{-27} \text{kg})}} \]

\[ = 1.17 \text{g} \text{ for proton possible} \]

The above is a "best of the envelope" estimate for experimental result. Every Cu, Al, and Be have been dropped.

![Graph showing range straggling data](image-url)
II. Stopping power

a) Electron capture and loss

b) Range straggling

\[ \text{electrons} \]

Notice: \[ \frac{R}{n} = \frac{1}{2} \left( \frac{M}{A m v} \right) \] \[ A = \text{aluminum} \]

2) Electron range straggling is huge to the point that the concept of electron range becomes vague.

There are several definitions of electron range:

1) Maximum range \((R)\) or \(R_{\text{max}}\)

   - The range is thus calculated by integrating over line average \(\text{d}E/\text{d}x\)

2) Practical range: defined by extrapolating the electron transmission curve to zero.

\[ \% \text{ of 100 electrons transmitted} \]

\[ R_0 \]

\[ 2 \times \left( \frac{E}{60^2} \right) \]
II. Stopping Power

d) Electron capture and loss

**Bohr Criterion:** "A rapidly moving nucleus is fully ionized if its velocity exceeds that of its most tightly bound electron."

The Bohr model:

$$E = \frac{m e^4}{8 \epsilon_0 h^2 n^2}$$

For innermost electron \((n=1)\):

$$\frac{1}{2}mv^2 = \frac{m e^4}{2(4\pi\epsilon_0)^2 h^2}$$

$$v = \frac{e}{4\pi\epsilon_0 h}$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0 h c} \quad \gamma = \frac{2e}{\alpha}$$

If \(\frac{v}{c} > \frac{e}{\alpha}\) then nucleus is fully ionized

Or if \(\frac{v}{c} = \frac{2e}{\beta} < \alpha\) the nucleus is fully ionized

If ion particle moving through material slows down such that \(\frac{v}{c} > \alpha\) then electrons may be captured by \(\frac{2e}{\beta}\) projectile, not by target.
II. Stopping Power

1) Electron capture and loss

Describing the charge state of your heavy ion projectile traveling through matter of a velocity below the solar criterion is very complicated.

There is a competition between electron capture and loss.

Accurate X-sections are needed to simulate the process.

Some insight into this process can be found using the Thomas-Fermi model to describe a nucleus \( \text{charged heavy projectile} \) moving slow enough so it has captured many electrons but fast enough so it's not neutral.

\[
\tilde{t}_{\text{eff}} = \text{effective charge of } = \frac{t}{1 - \tilde{\beta} c} \\
\text{projectile} \quad \text{emitted from projectile} \\
\tilde{\beta} c = \text{average } \# \text{ of captured electrons}
\]

For the purpose of simulations, you would like a relationship for \( \tilde{t}_{\text{eff}} \) in terms of \( \tilde{\beta} \) and \( t \).
II. Stopping Power

1) Electron Capture and Less

When $E < 10$ MeV, data indicates

$$\bar{\sigma} = \bar{\sigma} \left( 1 - e^{-\beta \sqrt{2/3}} \right)$$

where $\beta = 130 \pm 5$ \hspace{1cm} $\beta = \frac{V_{\text{projectile}}}{c}$

$\bar{\sigma}$ = a constant within a projectile

- When calculating stopping power for $E \geq 10$ MeV you use $\bar{\sigma}_{\text{free}}$ in Bethe-Blatt equation

Fluctuations in ion's charge state $\Rightarrow$ energy loss fluctuations

For thin absorbers (thickness of absorber is less than charge equilibrium distance defined as distance traveled until present half $\tau < \frac{2c}{\alpha}$)

For low energy radiation $\alpha < 1$, sample is thin

- Use stripping loss X-section

Thin Absorbers: The essentially determined expression is

$$\Delta \bar{\sigma}_{\text{eff}} = \frac{1}{2} \left( \bar{\sigma}_{\text{eff}} \left( 1 - \left[ \frac{2 \bar{\sigma}_{\text{eff}}}{\bar{\sigma}} \right]^{3/4} \right) \right)^{1/2}$$