MATH 4441/5541: Introduction to Numerical Analysis I

Instructor: Yunrong Zhu
Homework Assignment #1

Due Thursday September 5, 2013

1. Read Section 1.1 and do the following problems:

   (a) Show the equation $(x-2)^2 - \ln x = 0$ has at least one solution in the interval $[1, 2]$.

   (b) Find the absolute maximum and minimum of $f(x) = (1-e^x+2x)/3$ in the interval $[1, 2]$.

   (c) Suppose $f \in C[a, b]$ and $f'(x)$ exists on (a, b). Show that if $f'(x) \neq 0$ for all $x \in (a, b)$, then there can exist at most one number p in $[a, b]$ with $f(p) = 0$.

   (d) Find the second Taylor polynomial $P_2(x)$ for the function $f(x) = e^x \cos x$ about $x_0 = \pi/6$. Give a bound for the error $|f(x) - P_2(x)|$ if we used $P_2(x)$ as an approximation to $f(x)$ on the interval $[0, 1]$.

2. Read Section 2.1. Use Theorem 2.1 to find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-3} to the solution of $x^3 + x - 4 = 0$ lying in the interval $[1, 4]$. Find an approximation to the root with this degree of accuracy.

3. Read Section 2.3. Write *MATLAB* Programs for the Newton's method, Secant method and False Position method to find solution to the tolerance 10^{-5} for the equation $2x + 3\cos x - e^x = 0$ for $0 \leq x \leq 1$.