MATH 4441/5541: Introduction to Numerical Analysis I

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Homework Assignment #3

Due Thursday September 26, 2013

1. Read Section 3.1, and do the following:

   (a) Let $x_k = x_0 + kh$, for $k = 0, 1, 2, 3$, find $\max\limits_{x_0 \le x \le x_3} |L_{3,2}(x)|$.

   (b) Let $P_3(x)$ be the interpolating polynomial for the data $(0,0)$, $(0.5, y)$, $(1,3)$, and $(2,2)$. The coefficient of x^3 is 6. Find y.

2. Read Section 3.2. For a function f , the divided difference are given by

$x_0 = 0.0$	$f[x_0] =?$		
		$f[x_0, x_1] =?$	
$x_1 = 0.4$	$f[x_1] =?$		$f[x_0, x_1, x_2] = \frac{50}{7}$
		$f[x_1, x_2] = 10$	
$x_2 = 0.7$	$f[x_2] = 6$		

   Determine the missing entries in the table.

3. Write a MATLAB program to find the interpolating polynomial (in the Newton form) for a given data set (x_k, y_k), $\quad k = 0, \cdots, n$. Note, the polynomial can be represent uniquely by its coefficients. So we only need to output an array a_k for $k = 0, \cdots, n$ of the coefficients, and a way to evaluate the polynomial using the Newton's form for any other input x. To evaluate the polynomial, you may want to use the trick of "Nested Arithmetic" as demonstrated in Example 6 in Section 1.2 (Page 27).

   Use the program to test the following data set and get a polynomial of degree ≤ 8:

x_i	0	1	4	9	16	25	36	49	64
$f(x_i)$	0	1	2	3	4	5	6	7	8

   Compare your result with the Matlab subroutine `interp1` (piecewise linear interpolation by default – but you may choose other type of interpolation as well). Then compare it with the cubic spline interpolation by using the MATLAB subroutine `spline`:

```
y =0:8;
x =y.^(2);
xx = 0:0.5:64;
yy =spline(x, y, xx);
plot(x, y, 'r*', xx, yy);
```

From the results we get, which one is more accurate in the interval $[0, 64]$? Then compare these two interpolating polynomials in the interval $[0, 1]$ and see which one is more accurate. Type `help interp1` and `help spline` in Matlab for more details and examples of these subroutines.

4. Runge's Example: let $f(x) = \dfrac{1}{1+x^2}$ (Using the code in Problem 3).

   (a) Consider the interpolation of f in the interval $[-5, 5]$. Let $P_n(x)$ be the interpolating polynomial of degree $\leq n$ at the equally spaced interpolation points $x_i = -5 + \dfrac{10i}{n}$, $i = 0, \cdots, n$. Plot the error function $e(x) = f(x) - P_n(x)$ on the interval $[-5, 5]$ for each $n = 4, 8, 16, 32$, and recorder the approximate maximum of $|e(x)|$ for each n and where it occurs.

   (b) Repeat part (b) on the interval $[-3.6, 3.6]$ with the equally spaced interpolation points $x_i = -3.6 + \dfrac{7.2i}{n}$, $i = 0, \cdots, n$. What conclusion can you draw?

   (c) Repeat part (b) again on the interval $[-5, 5]$ with the interpolation points (the Chebyshev points):

$$x_k = 5\cos\left(\frac{(2k+1)\pi}{2n+2}\right), \quad k = 0, \cdots, n.$$

   What conclusion can you draw?